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Bound State Fermion Mass Formula for Supersymmetric Constituent Models

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Abstract

We calculate the "electromagnetic" mass shift of composite quasi Goldstone fermions, under the assumptions that supersymmetry is explicitly broken by gaugino mass term and that G/H is a symmetric space. It is shown that the fermion-boson mass splitting is not large. The result may be relevant for supersymmetric composite models as well as for supersymmetric technicolor models.

In the context of composite models, there has been growing interest in N=1 supersymmetric Yang-Mills theories. In the framework of such theories, the observed quarks and leptons are regarded as nearly massless bound states of preons, subject to confining strong hypercolor interactions (scale Λ_{HC}). The origin of their masslessness is a supersymmetric Goldstone mechanism [1]: The fermions (quasi Goldstone fermions, "QGF") are interpreted as superpartners of Goldstone bosons, originating from a spontaneous breakdown of a global internal group G to some subgroup H. For a variety of semi-realistic models, see [2].

Now, having achieved massless fermionic and bosonic bound states, the next step is to give them small ($\ll \Lambda_{HC}$) masses due to some symmetry perturbation. Normally, for non-Goldstone type bound states, one is unable to calculate the effect of a small mass generating perturbation, for one cannot solve the strong coupling hypercolor theory. However, the situation is special for Goldstone fields. The reason is that Goldstone fields have very peculiar dynamical ("PCAC") properties which do allow, in fact, the derivation of mass formulas which are valid basically non-perturbatively. For example using Dashen's formula [3] one can calculate the pion's electromagnetic mass shift even though one cannot yet solve the bound state problem in QCD.

Analogously, if we have massless fermionic bound states (QGF's) due to some supersymmetric Goldstone mechanism, we can calculate their mass using a supersymmetric version of Dashen's formula. In supersymmetric theories, there exist two Dashen type formulae [4-7] [FN1]:

$$(m_f)_{ij} = \frac{1}{f_{\pi_i} f_{\pi_j}} \langle 0 | [Q_i , [Q_j , \Delta L(0,\theta)|_{\theta=0}]] | 0 \rangle \quad (1)$$

$$(m_b^2)_{ij} = \frac{1}{f_{\pi_i} f_{\pi_j}} \langle 0 | [Q_i , [Q_j , \Delta L(0,\theta)|_{\theta\theta}]] | 0 \rangle \quad (2)$$

The indices i, j run over the broken global charges Q_i of G and ΔL is the perturbing chiral Lagrangian $(\bar{D}_{\dot\alpha} \Delta L = 0)$

$$\Delta L(x,\theta) = \frac{1}{2} \int d^2\bar\theta \, \Delta K(\phi,\bar\phi) + \Delta W(\phi) , \quad (3)$$

which is related to the total perturbating Lagrangian

$$\Delta \mathcal{L}(x) = \int d^2\theta \, \Delta L(x,\theta) + \int d^2\bar\theta \, \Delta \bar{L}(x,\bar\theta) . \quad (4)$$

The decay constants f_{π_i} are defined by the current field identity [7]

$$\langle 0 | J_i (x,\theta,\bar\theta) | \pi_j(q) \rangle = -i f_{\pi_i} \delta_{ij} \, e^{\theta \not{q} \bar\theta} e^{iq\cdot x} \quad (5)$$

where J_i is the broken current superfield. (1) can be derived using only QGF poles [4], so the mass on the L.H.S. is the QGF mass. Similar, (2) can be derived using only Goldstone boson poles, so the L.H.S. corresponds to the Goldstone boson mass^2. Of course, as long as supersymmetry is unbroken, $m_f = m_b$ and also the corresponding decay constants are equal.

There are now two ways to generate mass for composite Goldstone fields. First, one can explicitly break the group G by e.g. introducing some mass term in the superpotential $\Delta W(\phi)$, leaving supersymmetry unbroken. Note that a breaking term ΔK does not generate any mass. This is due to the fact that the Goldstone spectrum depends only on the properties of W and not on those of K [6-9], a direct consequence of the non-renormalization theorem [10]. Of course, leaving supersymmetry unbroken one cannot achieve a fermion-boson mass splitting as demanded by phenomenology. Thus, we prefer a second way of mass generation, that is, breaking both G and supersymmetry explicitly. This is particularly interesting since one knows that the coupling of a global supersymmetric theory to supergravity can induce explicit global supersymmetry breaking terms [11]. The general feature of such terms is that they are "soft" in the sense that they preserve the ultraviolet properties of the supersymmetric theory and that there is only a limited variety of such terms [11].

The most interesting soft breaking terms are mass terms given by

$$\Delta K = \mu^2 \theta^2 \bar{\theta}^2 \bar{\phi} \phi \tag{6}$$

$$\Delta W = \mu^2 \theta^2 \phi \phi \tag{7}$$

$$\Delta W = \frac{1}{16 g^2} \mu \theta^2 \, \mathrm{Tr} \, W^\alpha W_\alpha \tag{8}$$

Here, ϕ (W^{α}) denote generic chiral (gauge spinor) superfields in the linear Lagrangian. The supersymmetry violation is due to the explicit θ dependence. (6), (7) give mass contributions only to the scalar components of ϕ, while (8) represents a mass term for the gaugino. A typical value for μ is the gravitino mass; for our purposes only $\mu < \Lambda_{HC}$ is physically sensible.

In this paper, we discuss supersymmetry breaking by gaugino mass term (8). However, at the end of the paper we briefly comment also on (6), (7). To be specific, consider the following scenario: some global group G is spontaneously broken to a subgroup H, leading to a certain number of massless chiral Goldstone superfields. For simplicity, we assume that G/H is a symmetric space, i.e., the commutator of two broken generators gives only unbroken generators. Then the number of Goldstone superfields is necessarily dim (G/H) and not smaller [12]. That is, every Goldstone boson is accompanied by an additional scalar boson and one Weyl fermion.

Hence the Goldstone superfields are real with respect to H, thus allowing for mass generation. It follows also that the decay constants f_{π_i} are equal for the pion multiplet: $f_{\pi_i} = f_{\pi}$. The second assumption we make is that some subgroup $S \subseteq H$ is gauged. This is in complete analogy to nonsupersymmetric QCD where chiral $G = SU(2)_L \times SU(2)_R$ is broken to $H = SU(2)_V$ and $S = U(1)_{em}$ is gauged. The gauging of $U(1)_{em}$ breaks G explicitly, thus gives a radiative mass shift to the pions, which in lowest order is given by [13]

$$\delta m^2_{\pi^+} = \frac{3e^2}{f_\pi^2} \int \frac{d^4q}{(2\pi)^4} F(q^2) \tag{9}$$

where

$$\mathcal{F}(q^2) = \frac{1}{iq^2} \int dm^2 \, \frac{\rho_V(m^2) - \rho_A(m^2)}{q^2 + m^2 - i\epsilon} \quad . \tag{10}$$

Saturating the spectral functions ρ_V and ρ_A by the ρ and A_1 poles, one obtains [13]

$$\delta m^2_{\pi^+} = \frac{3\alpha}{2\pi} m_\rho^2 \ln 2 \ . \tag{11}$$

We want to repeat this calculation in the supersymmetric case, which nearby obtains also a mass formula for QGF. As stated above, the mass shift is zero as long as supersymmetry is intact. We shall assume that supersymmetry is broken explicitly by a mass term (8) for the S gaugino fields and that this is the only supersymmetry violating term. Because of this gauging, the kinetic term for the preons ϕ is modified to

$$\mathcal{L}(x) = \int d^4\theta \, \bar{\phi} \, e^{2g_{HC}V_{HC}} \, e^{2g\,V_S} \, \phi \quad , \tag{12}$$

where V_{HC}, V_S are the matrix valued hypercolor and S gauge superfields, respectively. Expanding $\mathcal{L}$ one gets to $O(g^2)$ the interaction of the S gauge fields with the preonic current superfields:

$$\mathcal{L}^{int}(x) = \int d^4\theta \left[2g\, V_S^a J_a + g^2 V_S^a V_S^b \tilde{J}_{\{a,b\}} + \cdots \right] \tag{13}$$

$$J_a = \bar{\phi}\, e^{2g_{HC}V_{HC}} S_a \, \phi$$

$$\tilde{J}_{\{a,b\}} = \bar{\phi}\, e^{2g_{HC}V_{HC}} \{ S_a, S_b \} \, \phi$$

Here the indices a,b run over the gauged generators S_a. The lowest order effective perturbing interaction at the composite level is then given by

$$\begin{aligned}\Delta L(x_1,\theta_1) &= \frac{1}{2}\int d^2\bar\theta_1\,\Delta K(x_1,\theta_1,\bar\theta_1)\\ &= g^2\int d^2\bar\theta_1\Big\{\int d^4x_2\,d^2\theta_2\,d^2\bar\theta_2\,\Delta_S^{ab}(\mu,1,2)\,T\,J_a(1)\,J_b(2)\\ &\quad -\frac{i}{2}\,\Delta_S^{ab}(\mu,1,1)\,\tilde J_{\{a,b\}}(1)\Big\}\,.\end{aligned}\tag{14}$$

$\Delta_S^{ab}(\mu,1,2)$ is the S gauge field propagator corresponding to the Lagrangian

$$\begin{aligned}\mathcal{L}^{kin}(x) &= \frac{1}{32g^2}\,Tr\left[\int d^2\theta\,(1+2\mu\theta^2)\,W_S^\alpha W_{S\alpha} + h.c.\right]\\ &\quad -\frac{1}{16\alpha}\,Tr\int d^2\theta\,d^2\bar\theta\,(D^2V_S)(\bar D^2V_S)\\ W_S^\alpha &= -\frac{1}{4}\bar D^2 e^{-2gV_S}D^\alpha e^{2gV_S}\,.\end{aligned}\tag{15}$$

It depends on the supersymmetry breaking parameter μ and is given in the Feynman gauge (α = 1) by [14]:

$$\Delta_S^{ab}(\mu,1,2) = \int\frac{d^4q}{(2\pi)^4}\,e^{-iq(x_1-x_2)}\,\tilde\Delta_S^{ab}(\mu,1,2,q^2)$$

$$\tilde\Delta_S^{ab}(\mu,1,2,q^2) = \frac{1}{q^2}\left[\Delta_A+\mu(\Delta_B+\bar\Delta_B)+\mu^2(\Delta_C+\bar\Delta_C)\right]\delta_{12}\,\delta^{ab}$$

$$\begin{aligned}\Delta_A &= -1\\ \Delta_B &= \frac{1}{8(q^2+\mu^2)}\,D^\alpha\bar D^2\theta^2 D_\alpha\\ \Delta_C &= \frac{-1}{32q^2(q^2+\mu^2)}\,D^\alpha\bar D^2\theta^2\bar\theta^2 D^2\bar D^{\dot\beta}\,\sigma_{\alpha\dot\beta}\\ \delta_{12} &= (\theta_1-\theta_2)^2(\bar\theta_1-\bar\theta_2)^2\end{aligned}\tag{16}$$

Putting (14) into (1) or (2) there appears a term involving $[Q_i, [Q_j, \langle \tilde{J} \rangle]]$. Now, there is a theorem [15] which states that if G/H is a symmetric space, no H singlet appears in $\{T_i, T_j\}$ other than δ_{ij}. Thus there is no contribution from $\tilde{J}$ since H is unbroken and we can effectively drop this term.

Consequently one gets [FN2]

$$(m_f)_{ij} = M_{ij}\Big|_{\theta=0,\, x=0} , \quad (m_b^2)_{ij} = \int d^2\theta \, M_{ij}\Big|_{x=0} , \tag{17}$$

where

$$M_{ij}(x_1, \theta_1) = \frac{g^2}{f_a^2} \int d^2\bar{\theta}_1 \, d^4x_2 \, d^2\theta_2 \, d^2\bar{\theta}_2 \; \Delta_S^{ab}(p, 1, 2) \cdot$$
$$\cdot \left[Q_i, \left[Q_j, \langle 0 | T \, J_a(1) \, J_b(2) | 0 \rangle \right]\right] \tag{18}$$

Using again that G/H is a symmetric space, the double commutator term can be simplified [15]

$$\delta^{ab} \left[Q_i, \left[Q_j, \langle 0 | T \, J_a(1) J_b(2) | 0 \rangle \right]\right]$$
$$= \delta_{ij} \, C_S^2(r_i) \langle 0 | T \{ J^T(1) J^T(2) - J^X(1) J^X(2) \} | 0 \rangle \tag{19}$$

In this notation, $J^X(J^T)$ denotes any single broken (unbroken) current (no sum), and $C_S^2(r_i)$ is the quadratic Casimir invariant of S of the representation r_i under which the broken generator X_i transforms.

To proceed, we need the superspace spectral decomposition of the current correlation functions, involving the longitudinal ($P_L + \bar{P}_L$) and transverse (P_T) projectors:

$$\int d^4x_2\, e^{-iqx_2} \langle 0 | T\, J^T(0,\theta_1,\bar\theta_1)\, J^T(x_2,\theta_2,\bar\theta_2) | 0 \rangle$$

$$= \frac{1}{i} \int dm^2\, \rho_T(m^2) \left[\frac{P_T}{q^2+m^2-i\epsilon} + \frac{P_L+\bar{P}_L}{m^2} \right] \delta_{12} \qquad (20a)$$

$$= \frac{1}{i} \int dm^2\, \frac{\rho_T(m^2)}{q^2+m^2-i\epsilon} \left[1 - \frac{1}{16m^2} \{D^2, \bar{D}^2\} \right] \delta_{12}$$

ρ_T is the corresponding spectral function and we used $P_T = 1 - P_L - \bar{P}_L$ as well as $P_L = 1/16\,\Box^{-1}\bar{D}^2D^2$. (20a) is determined by the requirement that <JJ> turns into a massive vector propagator if one saturates the spectrum with a single pole. The axial current correlation function contains in addition a pole term due to the Goldstone excitation, which can be derived from (5):

$$\int d^4x_2\, e^{-iqx_2} \langle 0 | T\, J^X(0,\theta_1,\bar\theta_1)\, J^X(x_2,\theta_2,\bar\theta_2) | 0 \rangle \qquad (20b)$$

$$= \frac{1}{i} \left\{ \int dm^2\, \frac{\rho_X(m^2)}{q^2+m^2-i\epsilon} \left[1 - \frac{1}{16m^2} \{D^2, \bar{D}^2\} \right] - \frac{1}{q^2-i\epsilon} \frac{f_\pi^2}{16} \{D^2, \bar{D}^2\} \right\} \delta_{12}$$

Thus
$$M_{ij}(0,\theta_1) = \delta_{ij}\, C_S^2(r_i)\, \frac{g^2}{f_\pi^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{iq^2} \int d^2\bar\theta_1\, d^2\theta_2\, d^2\bar\theta_2 \cdot$$
$$\cdot \Big\{ \big[\Delta_A + \mu(\Delta_B + \bar\Delta_B) + \mu^2(\Delta_C + \bar\Delta_C))\delta_{12} \big] \cdot$$
$$\cdot \Big[\int dm^2 \; \frac{\rho_T(m^2) - \rho_X(m^2)}{q^2 + m^2 - i\epsilon} \Big(1 - \frac{1}{16m^2}\{D^2, \bar D^2\} \Big) + \frac{f_\pi^2}{16} \frac{\{D^2, \bar D^2\}}{q^2 - i\epsilon} \Big] \delta_{12} \Big\} \qquad (21)$$
.

Performing the superspace integrals it turns out that the $\{D^2, \bar D^2\}$, Δ_A and Δ_B terms do not contribute:

$$M_{ij}(0,\theta_1) = \delta_{ij}\, C_S^2(r_i)\, \frac{g^2}{f_\pi^2} \int \frac{d^4q}{(2\pi)^4}\, F(q^2)\, G(q^2, \theta_1) \,, \qquad (22)$$

where $F(q^2)$ is defined in (10) and

$$G(q^2,\theta_1) = \int d^2\bar\theta_1 \Big((\mu \bar\Delta_B + \mu^2(\Delta_C + \bar\Delta_C))\delta_{12}\big|_{2=1} \Big) \qquad (23)$$

For the fermions and bosons different θ-components and therefore Δ's contribute; in detail one obtains for the fermions

$$G(q^2, 0) = \int d^2\bar\theta_1 \big(\mu \bar\Delta_B\, \delta_{12}\big|_{2=1} \big) = 2\mu \frac{1}{q^2 + \mu^2} \,, \qquad (24)$$

while for the bosons

$$G(q^2,\theta_1)\Big|_{\theta_1\bar\theta_1} = \int d^2\theta_1\, d^2\bar\theta_1 \big(\mu^2(\Delta_C + \bar\Delta_C)\delta_{12}\big|_{2=1} \big) \qquad (25)$$
$$= 4\mu^2 \frac{1}{q^2+\mu^2} \,.$$

Hence we finally arrive at

$$(m_f)_{ij} = \frac{1}{2}\mu\, \delta_{ij}\, C_S^2(r_i)\, A \,, \quad (m_b^2)_{ij} = \mu^2 \delta_{ij}\, C_S^2(r_i)\, A \tag{26}$$

with

$$A = \frac{4g^2}{f_\pi^2}\int \frac{d^4q}{(2\pi)^4}\, F(q^2)\, \frac{1}{q^2+\mu^2} \tag{27}$$

Clearly, for vanishing supersymmetry breaking $\mu \to 0$ all masses vanish. For $\mu \to \infty$ the bosonic formula turns into the non-supersymmetric one (9), but with a factor 4 is front instead of 3. This is due to the additonal "D" contribution (weight 1) besides that of the vector boson (weight 3). However, in the fermionic sector $\mu \to \infty$ is unphysical: $m_f \to 0$. That is, only $\mu < \Lambda_{HC}$ is sensible since otherwise there is no reason for QGF states to appear.

The factor A depends on details of the excited spectrum contributing to the current correlation functions. As an approximation one can saturate <JJ> by the poles of the analogues of the ρ and A_1 vector mesons:

$$\rho_T(m^2) = g_\rho^2\, \delta(m^2 - m_\rho^2) \,, \quad \rho_X(m^2) = g_A^2\, \delta(m^2 - m_A^2) \tag{28}$$

where $g_\rho^2 = g_A^2$ because of Weinberg's sume rule [16]. Assuming that $g_\rho^2 = 2m_\rho^2 f_\pi^2$ as in the non-supersymmetric case, we finally obtain, evaluating the momentum integral ($\alpha = \frac{g^2}{4\pi}$)

$$A = \frac{2\alpha}{\pi} m_S^2 \left[\frac{1}{m_S^2 - \mu^2} \ln\left(\frac{m_S^2}{\mu^2}\right) - \frac{1}{m_A^2 - \mu^2} \ln\left(\frac{m_A^2}{\mu^2}\right) \right] . \tag{29}$$

We see that m_f, $m_b \to 0$ if the mass splitting in the heavy sector goes to zero: $\Delta M = m_A - m_S \to 0$. That is, ΔM is a measure of chiral symmetry breaking in the heavy sector. It is well known that if some chiral symmetry is unbroken so that $\Delta M = 0$, m_f remains zero under radiative corrections. The interesting point here is that ΔM controls also the bosonic sector (just as in the non-supersymmetric case). This means, one cannot achieve a large fermion-boson mass splitting by demanding that the amount of chiral symmetry breaking in the heavy sector is small. Put in other words, if some chiral symmetry (e.g. $U(1)_X$) is unbroken so that it protects the QGF to acquire radiative mass, it protects also the bosons.

For m_S , ΔM , $\mu = O(\Lambda_{HC})$ one has typically $A = \alpha\, O(1)$. Thus, the fermion-boson mass ratio is roughly

$$m_f / m_b = \alpha^{1/2} \tag{30}$$

i.e. not very large.

Our calculation was based on the assumption that G/H is a symmetric space. It would be interesting to investigate which of the above-discussed features apply also to the general case. Of course, the formulas (26) are not of general use because they contain unknown parameters. Furthermore, in a realistic model also other explicit supersymmetry breaking terms may occur, e.g. of type (6) or (7). In addition f_π and the spectral functions need not coincide in the fermionic and bosonic sectors if supersymmetry is explicitly broken. The effect of the interplay of all these supersymmetry breakings is hard to calculate.

Finally, we comment briefly on the fermion-boson splitting induced by breaking terms like (6) or (7) [FN3]. ((6) splits in addition the two scalar degrees of freedom, while (7) does not). Typically

$$
\begin{aligned}
(m_f)_{ij} &= \frac{1}{f_{\pi_i} f_{\pi_j}} \left[Q_i, \left[Q_j, \mu^2 \theta^2 \langle \phi\phi \rangle \big|_{\theta=0} \right]\right] = 0 , \\
(m_b^2)_{ij} &= \frac{1}{f_{\pi_i} f_{\pi_j}} \left[Q_i, \left[Q_j, \mu^2 \langle \phi\phi \rangle \right]\right] \neq 0 .
\end{aligned}
\qquad (31)
$$

Thus, inspite of explicit G and supersymmetry breaking the fermions remain massless. However, one may expect that the fermions acquire mass due to higher order radiative corrections. Perhaps this mechanism allows to construct realistic models with large fermion-boson mass splittings.

Note added: After completing the manuscript, we received the preprint [17] where also soft supersymmetry breaking terms are discussed.

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Footnotes

[FN1] Inspite of that (2) involves a F-Term on the R.H.S., it is not correct to conclude that $m_b^2 = 0$ if supersymmetry is unbroken [5].

[FN2] The real and imaginary scalar components remain degenerate because the S gauge interactions do not distinguish between them.

[FN3] We assume that these terms do not change the pattern of symmetry breaking. This may easily occur because in supersymmetric gauge theories the effective potential has degenerate directions, thus is very sensitive to small perturbations.