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Effective Lagrangian and Mass Generation in Supersymmetric QCD

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We calculate the electromagnetic mass shift of composite Goldstone bosons and quasi-Goldstone fermions in supersymmetric QCD for the case where supersymmetry is explicitly broken by a photino mass term. We use an effective Lagrangian approach incorporating the analogues of the ρ and A_1 vector mesons together with vector meson dominance.

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In the framework of composite models, quarks and leptons are regarded as (nearly) massless bound states of preons. In a specific approach [1], the origin of their masslessness is a supersymmetric Goldstone mechanism: The observed fermions are interpreted as N=1 supersymmetric partners (quasi-Goldstone-fermions, "QGF") of Goldstone bosons, arising from a spontaneous breakdown of a global internal symmetry G to some subgroup H. The observed finite mass shifts are due to small symmetry perturbations.

Of course, to achieve a realistic mass spectrum, supersymmetry has to be broken. This can be done either spontaneously or explicitly. The latter possibility is particular intriguing since the coupling to supergravity leads to explicit breaking terms quite naturally [2]. Such breaking terms are soft and of well defined general form, so one can determine their effects on the mass spectrum quite easily [3].

A detailed analysis based on effective Lagrangian method has been presented in ref. [4]. On the other hand, in ref. [5] the masses of QGF have been computed in complete analogy to the electromagnetic pion mass shift calculation in non-supersymmetric QCD [6]. Supersymmetry was broken explicitly only by photino mass term. The advantage of this approach is that it takes the heavy excited spectrum into account. In [5], the masses of charged Goldstone bosons and QGF are given in terms of the masses of the ρ and A_1 vector mesons

$$m_B^2 = \mu^2 A(\mu^2) \quad , \qquad m_F = \mu/2 \; A(\mu^2) \tag{1}$$

where

$$A(\mu^2) = \frac{2\alpha}{\pi}\left(\frac{m_\rho^2}{m_\rho^2-\mu^2}\ln\left(\frac{m_\rho^2}{\mu^2}\right) - \frac{m_\rho^2}{m_A^2-\mu^2}\ln\left(\frac{m_A^2}{\mu^2}\right)\right). \qquad (2)$$

Here μ is the supersymmetry breaking photino mass parameter, and it was implicitly assumed $m_A^2 = 2m_\rho^2$. In the nonsupersymmetric limit $\mu \longrightarrow \infty$ with $m_A^2 = 2m_\rho^2$ one obtains

$$m_B^2 = 4\frac{\alpha}{2\pi}\, m_\rho^2 \ln 2 \quad , \qquad (3)$$

which is precisely the conventional electromagnetic pion mass shift as derived in Refs. [6,7], except the factor 4 instead of 3. (This is due to an additional supersymmetric contribution).

Now, the results (1) have been obtained by using the two supersymmetric versions [8] of Dashen's formula [9], which are valid even for nonperturbative bound states. However, in contrast to the bosonic case, the derivation of the fermionic Dashen's formula requires unbroken supersymmetry. That is there could in principle appear additional contributions to m_F.

The purpose of this paper is to support the results (1) by calculating m_B^2 and m_F using a supersymmetric generalization of the effective Lagrangian approach [7,10]. That is we compute the masses perturbatively using an effective Lagrangian containing the chiral Goldstone superfields Π as well as the corresponding ρ and A_1 vector meson superfields.

As toy model we consider two flavor SQCD with $G = SU(2)_L \times SU(2)_R$. We assume G is broken to $H = SU(2)_V$ while $U(1)_{em} \subset SU(2)_V$ is gauged. Supersymmetry is explicitly broken by a photino mass term.

To construct the effective Lagrangian we follow the approach of Ref. [11]. The nonlinear action for the chiral Goldstone superfields $\Pi = \Pi_a \tau_a$, a = 1 ... 3 alone is given by [12]

$$\Gamma = \frac{1}{2} f_\pi^2 \, \mathrm{Tr} \int d^8 z \; M^+ M \tag{4}$$

where $M = \exp\left(i \sqrt{2}\, \Pi / f_\pi\right)$.

The global $SU(2)_L$ current superfields J_L are defined by the noninvariance of the SQCD action under an infinitesimal local $SU(2)_L$ transformation with chiral superparameters Λ_L:

$$\delta_{\Lambda,\bar\Lambda} \, \Gamma^{SQCD} = i \, \mathrm{Tr} \int d^8 z \, (\Lambda_L - \bar\Lambda_L) \, J_L \tag{5}$$

(The same applies for J_R too). The currents transform as

$$J_L' = e^{-i\bar\Lambda_L} J_L \, e^{i\Lambda_L} \, , \quad J_R' = e^{-i\Lambda_R} J_R \, e^{i\bar\Lambda_R} \, , \tag{6}$$

while the Goldstones obey

$$M = e^{-i\Lambda_R} M \, e^{i\Lambda_L} \, . \tag{7}$$

We like to include J_L and J_R as dynamical fields in our effective action Γ^{eff}. The constraint is that Γ^{eff} should obey the same local Ward identity (5). Since the currents do not transform like gauge fields, we redefine variables according to the supersymmetric generalization of the current field identities [11]

$$J_L = \frac{m^2}{g^2}\, e^{gV_L} \quad , \quad J_R = -\frac{m^2}{g^2}\, e^{-gV_R} \quad . \tag{8}$$

Here m is some scale, g^2 the effective strong coupling constant and V_L,V_R are left and right handed gauge fields (The negative value of $\langle J_R \rangle$ follows from (5) and (6)). Hence one gets as simplest effective action (the factor $(1-\beta^2)^{-1}$ is explained below)

$$\Gamma^{eff} = \Gamma^{gauge}_{inv.} + \Gamma^{break}$$

$$\Gamma^{gauge}_{inv.} = \left(\frac{1}{4g^2} Tr \int d^6z \left(W^\alpha(V_L) W_\alpha(V_L) + W^\alpha(V_R) W_\alpha(V_R) \right) + h.c. \right)$$
$$+ \frac{1}{2} f_\pi^2 \left(\frac{1}{1-\beta^2} \right) Tr \int d^8z\, M^+ e^{gV_R} M\, e^{-gV_L} \tag{9}$$

$$\Gamma^{break} = \frac{m^2}{g^2} Tr \int d^8z \left(e^{gV_L} + e^{-gV_R} \right) \quad .$$

Here $W^\alpha(V_L)$, $W^\alpha(V_R)$ are the usual chiral fields strength spinors corresponding to V_L and V_R. Γ^{eff} is invariant under global $SU(2)_L \times SU(2)_R$ transformations. The non-invariance of Γ^{break} under local $SU(2)_L \times SU(2)_R$ transformations reproduces precisely (5) by use of (8).

Γ^{eff} describes the interactions of Goldstone superfields with vector superfields of mass $\frac{1}{2}m^2$ as generalization of the non-supersymmetric case. However, Γ^{eff} involves in addition the scalar components of the superfields which feel the usual "D" -potential V. To assure a supersymmetric ground state V=0 at $\langle V_L \rangle = \langle V_R \rangle = \langle \pi \rangle = 0$ we derive as constraint

$$\frac{1}{2} f_\pi^2 \frac{1}{1-\beta^2} = \frac{m^2}{g^2} \tag{10}$$

This relation seems model dependent and is due to our specific choice of Γ^{eff}. Therefore we treat in the following m and f_π still as independent.

Next define

$$J_V = \frac{1}{2}\left(J_L + J_R\right), \quad J_A = \frac{1}{2}\left(J_L - J_R\right) \tag{11}$$

as well as supersymmetric analogues of the ρ and A_1 vector mesons

$$\rho = \frac{1}{2}\left(V_L + V_R\right), \quad A' = \frac{1}{2}\left(V_L - V_R\right). \tag{12}$$

This implies the current field identities between J_V and ρ as well as J_A and A' are rather complicated:

$$\begin{aligned} J_V &= \frac{1}{2}\frac{m^2}{g^2}\left(e^{g(\rho+A')} - e^{-g(\rho-A')}\right) = \frac{m^2}{g^2}\left(g\rho + \frac{1}{2}g^2\{A',\rho\} + \cdots\right) \\ J_A &= \frac{1}{2}\frac{m^2}{g^2}\left(e^{g(\rho+A')} + e^{-g(\rho-A')}\right) = \frac{m^2}{g^2}\left(1 + gA' + \frac{1}{2}g^2(\rho^2 + A'^2) + \cdots\right) \end{aligned} \tag{13}$$

implying $\langle J_V\rangle = 0$ and $\langle J_A\rangle = \frac{m^2}{g^2}$. In the Wess-Zumino gauge, (13) contains the conventional non-supersymmetric current field identities as the $\theta\bar{\theta}$ components.

Substituting the redefined fields (12) back into Γ^{eff} there occurs a mass mixing term

$$\Gamma^{mix} = i \frac{g\sqrt{2} f_\pi}{1-\beta^2} \, Tr \int d^8 z \, (\bar{\pi} - \pi) A' . \tag{14}$$

Diagonalization leads to redefine

$$A' = A + i \frac{\beta}{\sqrt{2}\, m} (\pi - \bar{\pi}) , \tag{15}$$

which introduces additional Goldstone-vector interactions also in the kinetic terms. The $(1-\beta^2)^{-1}$ factor in Γ^{eff} secures that the Goldstone fields have canonical kinetic terms after this diagonalization, if

$$\beta = \frac{g f_\pi}{m} . \tag{16}$$

The masses of the vector mesons are now

$$m_S^2 = m^2 , \qquad m_A^2 = m^2 \frac{1}{1-\beta^2} \tag{17}$$

To achieve a $\pi_+ - \pi_0$ mass shift, electromagnetism has to be included. This is done substituting $Tr(J_L - J_R)$ in (9) by

$$\begin{aligned} & Tr\left(J_L e^{eV\tau_3} - J_R e^{-eV\tau_3} \right) \\ &= \frac{m_S^2}{g^2} Tr\left(e^{g(S+A')} e^{eV\tau_3} + e^{-g(S-A')} e^{-eV\tau_3} \right) \end{aligned} \tag{18}$$

which now breaks in addition global $SU(2)_V$ invariance. The expansion of Γ^{break} contains a mixing term

$$2\,\frac{e}{g}\,m_s^2 \int d^8z \;\rho^0 V \tag{19}$$

just as the non-supersymmetric case. Of course, due to this mixing neither ρ^0 nor V correspond to the physical particles. However, for lowest order processes it suffices to take as electromagnetic interaction only (19). This is precisely the idea of ρ vector dominance.

Now, in order to generate mass at all one has to break supersymmetry. This is done by a photino mass term parametrized by μ. Hence one adds

$$\begin{aligned}\Gamma^{em} = {} & \frac{1}{4e^2}\left(\int d^6z\,(1+2\mu\theta^2)\,W^\alpha(V)W_\alpha(V) + h.c.\right) \\ & - \frac{1}{16}\int d^8z\,(D^2V)(\bar{D}^2V)\quad .\end{aligned} \tag{20}$$

Supersymmetry is violated due to the explicit θ dependence. Γ^{em} leads to the photon propagator [13]

$$i\Delta^\gamma(1,2,q^2,\mu) \;=\; i\Delta_0^\gamma(1,2,q^2) + i\tilde{\Delta}^\gamma(1,2,q^2,\mu) \tag{21}$$

where
$$\begin{aligned}
i\Delta_0^\gamma(1,2,q^2) &= -\frac{1}{q^2}\,\delta_{12} \\
i\tilde{\Delta}^\gamma(1,2,q^2,\mu) &= \frac{1}{q^2}\left(\mu\,(\Delta_B+\bar{\Delta}_B) + \mu^2(\Delta_C+\bar{\Delta}_C)\right)\delta_{12} \\
\Delta_B &= \frac{1}{8(q^2+\mu^2)}\,D^\alpha\bar{D}^2\theta^2 D_\alpha \\
\Delta_C &= \frac{-1}{32\,q^2(q^2+\mu^2)}\,D^\alpha\bar{D}^2\theta^2\bar{\theta}^2 D^2\bar{D}^{\dot\beta}\,q_{\alpha\dot\beta} \\
\delta_{12} &= (\theta_1-\theta_2)^2(\bar{\theta}_1-\bar{\theta}_2)^2\quad .
\end{aligned}$$

Δ^{γ}_{o} represents the supersymmetry preserving part; its contribution vanishes because of the non-renormalization theorem [14] . Therefore we omit Δ^{γ}_{o}, i.e. we set $\Delta^{\gamma} = \tilde{\Delta}^{\gamma}$. For the A propagator we use

$$i\Delta^{A}_{ab}(1,2,q^2) = \frac{1}{q^2 + m_A^2}\left(1 - \frac{\{D^2, \bar{D}^2\}}{16\, m_A^2}\right)\delta_{12}\,\delta_{ab} \tag{22}$$

while for ρ we take an effective propagator reflecting vector dominance

$$i\Delta^{\rho\gamma\rho}_{ab}(1,2,q^2,\mu) = -i\Delta^{\rho}\tilde{\Delta}^{\gamma}\Delta^{\rho}\delta_{a3}\,\delta_{b3} = -\frac{e^2}{g^2}i\left(\frac{m_\rho^2}{q^2 + m_\rho^2}\right)^2\tilde{\Delta}^{\gamma}\delta_{a3}\,\delta_{b3} \tag{23}$$

which is still transverse:

$$\left(\frac{D^{\alpha}\bar{D}^2 D_{\alpha}}{8\, q^2}\right)\Delta^{\rho\gamma\rho} = \Delta^{\rho\gamma\rho} \quad , \quad D^2\Delta^{\rho\gamma\rho} = \bar{D}^2\Delta^{\rho\gamma\rho} = 0 \tag{24}$$

Now we are in the position to calculate the pion's mass shift using ρ -dominance. That is we calculate

$$\Gamma^{mass} = \int d^6 z\; \Pi_+\Pi_-\; M(x,\theta) \;+\; h.c. \tag{25}$$

by considering all tree diagrams for $\rho^{o}\pi \to \rho^{o}\pi$ and closing the $\rho^{o}\gamma\rho^{o}$ loop with the propagator (23). Then the bosonic and fermionic masses are given by

$$m_B^2 = \int d^2\theta\; M(0,\theta) \;, \quad m_F = M(0,\theta = 0) \;. \tag{26}$$

(we do not consider here bosonic mass terms of the form $\overset{*}{\pi}\pi$).

The relevant vertices contributing to M contained in Γ^{eff} are of the form $\pi\bar{\pi}\rho$, $\pi\bar{\pi}\rho\rho$, $\pi\bar{\pi}\rho D^{\alpha}\bar{D}^{2}D_{\alpha}\rho$, $\pi A D^{\alpha}\bar{D}^{2}D_{\alpha}\rho$ and permutations. Owing to the transversality of $\Delta^{\rho\gamma\rho}$ (24) many terms like the $\{D^2,\bar{D}^2\}$ part of Δ^A vanish. The contributing diagrams are displayed in the figure. They all have the same θ-structure. In detail one obtains

$$M(0,\theta) = \int \frac{d^4q}{(2\pi)^4} F(q^2)\, G(q^2,\theta) \tag{27}$$

where

$$F(q^2) = \frac{e^2 m_\rho^4}{i(q^2+m_\rho^2)^2}\left(\frac{1}{q^2} + \frac{\beta^2}{m_\rho^2} - \frac{\beta^2}{m_\rho^2}\,\frac{q^2}{q^2+m_A^2}\right), \tag{28}$$

and $G(q^2, \theta)$ is basically the photon propagator closed in θ-space

$$G(q^2,\theta_1) = \int d^2\bar{\theta}_1\, \tilde{\Delta}^{\gamma}(1,2,q^2,\mu)\big|_{2=1} . \tag{29}$$

Since [5]

$$\begin{aligned} \int d^2\theta\, G(q^2,\theta) &= 4\mu^2 \frac{1}{q^2+\mu^2} \\ G(q^2,0) &= 2\mu \frac{1}{q^2+\mu^2} \end{aligned} \tag{30}$$

one computes from (27)

$$m_B^2 = \mu^2 A(\mu^2) \quad , \quad m_F = \tfrac{1}{2}\mu A(\mu^2) \tag{31}$$

with

$$A(\mu^2) = 4 \int \frac{d^4q}{(2\pi)^4} F(q^2) \frac{1}{q^2+\mu^2} \tag{32}$$
$$= \frac{\alpha}{\pi}\left(\frac{m_A^2}{m_A^2-m_\rho^2}\right)\left(\frac{m_\rho^2}{m_\rho^2-\mu^2}\ln\left(\frac{m_\rho^2}{\mu^2}\right) - \frac{m_\rho^2}{m_A^2-\mu^2}\ln\left(\frac{m_A^2}{\mu^2}\right)\right) .$$

In the non-supersymmetric case β^2 is a free parameter which is empirically found to be $\beta^2 = 1/2$, implying $m_A^2 = 2m_\rho^2$. Hence we recover precisely the results (1) of ref. [5] where to saturate the spectral functions $m_A^2 = 2m_\rho^2$ has been assumed implicitly. That is, the use of the fermionic Dashen's formula is justified in this case, though supersymmetry is broken explicity.

On the other hand in the effective Lagrangian (9) parameters are constrained by (10) in order that the potential vanishes. This fixes $\beta^2 = 2/3$ and leads to $m_A^2 = 3m_\rho^2$. However, as remarked above, this relation may be model dependent.

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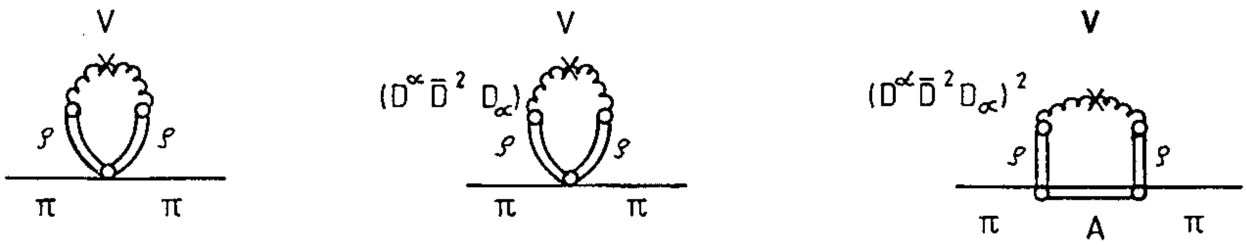
Figure Caption:

Supergraph contributions to M. The cross denotes the supersymmetry breaking photino mass.



References

[1] W. Buchmüller, S.T. Love, R.D. Peccei and T. Yanagida, Phys.Lett. 115B (1982) 233.

[2] L. Girardello and M.T. Grisaru, Nucl.Phys. B194 (1982) 65; R. Barbieri, S. Ferrara and C. Savoy, Phys.Lett. 119B (1982) 343.

[3] D. Lüst, Nucl.Phys. B239 (1984) 508; W. Lerche and D. Lüst, Nucl.Phys. B244 (1984) 157.

[4] A. Masiero and G. Veneziano, "Split light composite supermultiplets", CERN preprint CERN-TH-3950/84.

[5] W. Lerche, Phys.Lett. 149B (1984) 361.

[6] T. Das, G. Guralnik, V. Mathur, F. Low and J. Young, Phys.Rev.Lett. 18 (1967) 759.

[7] G.C. Wick and B. Zumino, Phys.Lett. 25B (1967) 479.

[8] G. Veneziano, Phys.Lett. 128B (1983) 199;
G.M. Shore, Nucl.Phys. B231 (1984) 139;
T. Clark and S. Love, Nucl.Phys. B232 (1984) 306
W. Lerche, Nucl.Phys. B246 (1984) 475.

[9] R.F. Dashen, Phys.Rev. 183 (1969) 1245, ibid D3 (1971) 1879.

[10] B.W. Lee and H.T. Nieh, Phys.Rev. 166 (1967) 1507.

[11] K. Konishi, "Dynamical Supersymmetry Breaking and Effective Lagrangian in an SU(5)-Model", Pisa preprint IFUP TH-11/84 (1984); E. Guadagnini and K. Konishi, "Effective Gauge Symmetry in Supersymmetric Confining Theories", Pisa preprint (1984).

[12] W. Lerche, Nucl.Phys. B238 (1984) 582.

[13] J. Helayël-Neto, Phys.Lett. 135B (1984) 78.

[14] W. Lerche, R.D. Peccei and V. Višnjič, Phys.Lett. 140B (1984) 363.