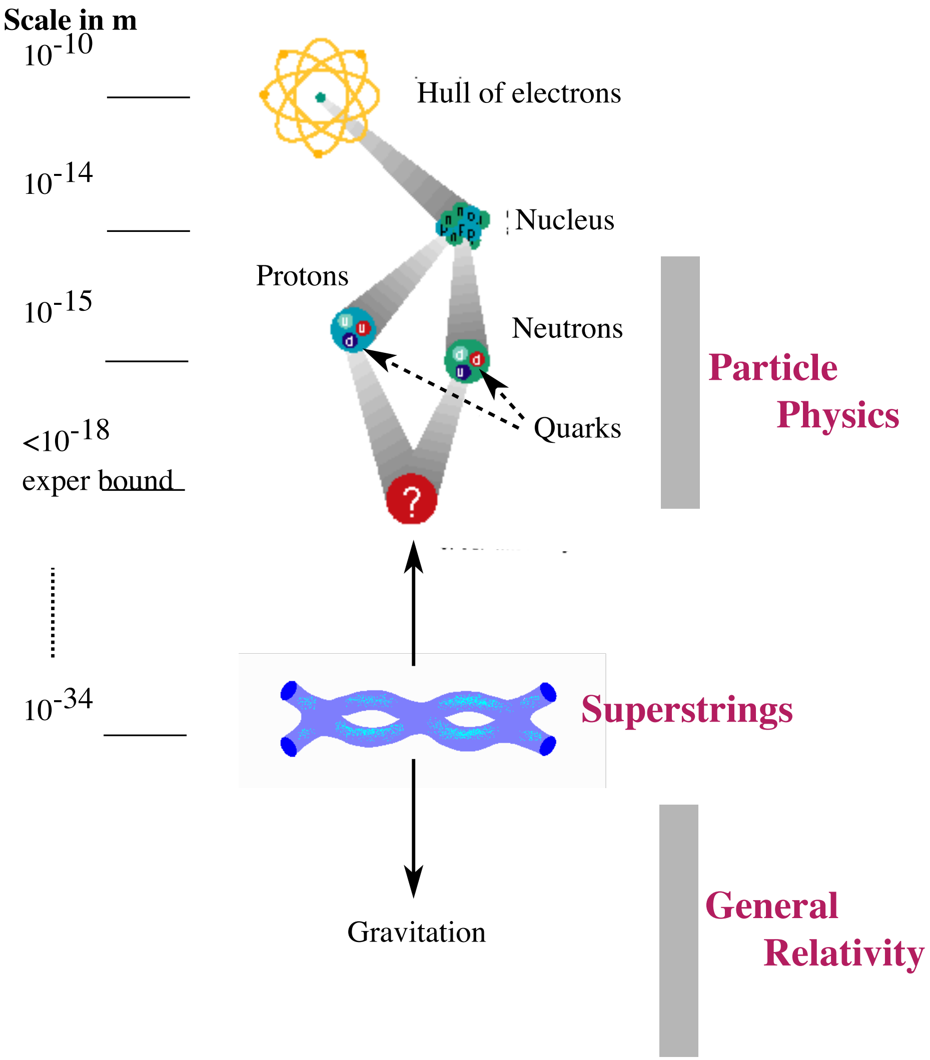
An Overview of String Theory

W. Lerche, ISTAPP 2011
Istanbul Part 1

- **Perturbative string theories**
  - Motivation: the Standard Model and its Deficiencies
  - String theory as 2d conformal field theory
  - Consistency conditions on string constructions
  - Bosonic and supersymmetric strings in D=26,10
- **Compactification to lower dimensions**
  - T-Duality, minimal length scales
  - Supersymmetry, geometry and zero modes
  - Parameter spaces, geometrization of coupling constants
  - Stringy predictions ?
- **Non-perturbative string dualities**
  - Non-perturbative quantum equivalences
  - S-Duality in SUSY gauge theories
  - D-branes and Stringy Geometry
  - Unification of string vacua
- **Tests and Applications**
  - "Theoretical experiments": tests and consistency checks
  - D-brane approach to QFT
  - Recent developments

Fundamental Structure of Matter



Scale in m

10^{-10}

10^{-14}

10^{-15}

$<10^{-18}$
exper bound

10^{-34}

Hull of electrons

Nucleus

Protons

Neutrons

Quarks

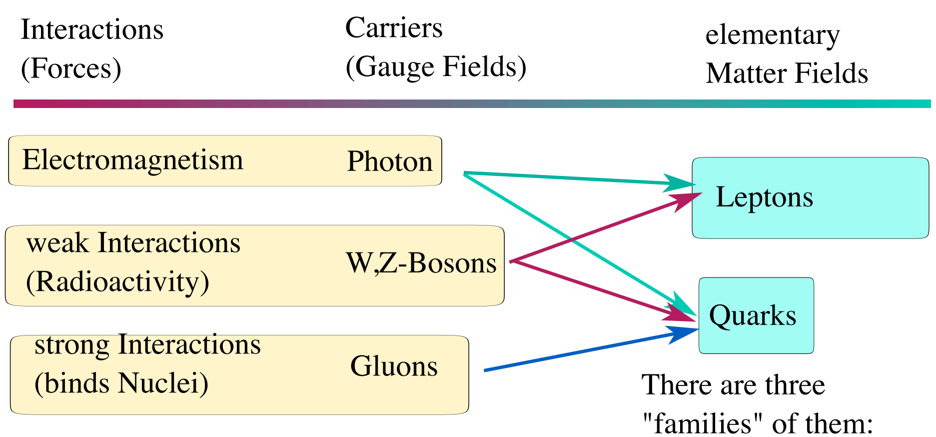
Particle Physics

Superstrings

Gravitation

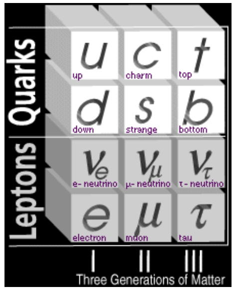
General Relativity

Physics of Elementary Particles

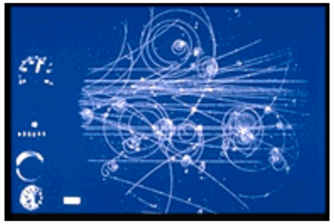


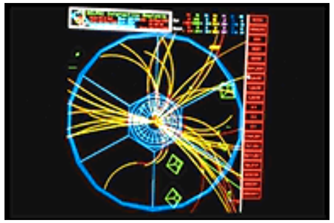
Interactions (Forces)	Carriers (Gauge Fields)	elementary Matter Fields
Electromagnetism	Photon	Leptons, Quarks
weak Interactions (Radioactivity)	W,Z-Bosons	Leptons, Quarks
strong Interactions (binds Nuclei)	Gluons	Quarks

There are three "families" of them:



The "Standard-Model" of perticle physics describes subnuclear phenomena with partly stunning accuracy !



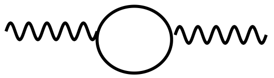


Deficiencies of the Standard Model

- Its structure is quite ad hoc: are there deeper principles ?
  ("**grand unification**" of all matter and forces)
- ca 25 **free parameters**: determined by what ?

$$\mathcal{L} = \left(\sum \bar\psi\gamma(\partial + \underset{\substack{\uparrow \\ \text{gauge couplings}}}{g_k} A)\psi\right) + \left(\sum \underset{\substack{\uparrow \\ \text{masses}}}{m_i}\bar\psi\psi + \underset{\substack{\uparrow \\ \text{Higgs VEVs}}}{\varphi_j}\bar\psi\psi\right) + \dots$$

- **Gauge hierarchy**:
  why weak scale (100GeV) << Unification scale scale (10^16GeV)
- Instability of parameters through self-interactions



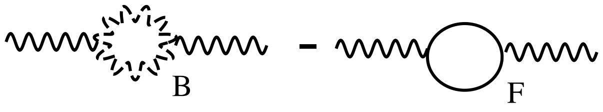
$= \infty$

(Renormalization:
large scale hierarchies problematic)

"Supersymmetry"

Symmetry between bosons and fermions

improved divergence structure:

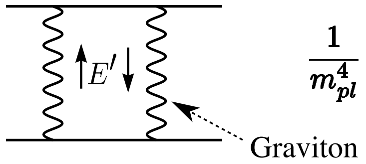


$= 0$

Problems of Quantum Gravity

- The Standard-Model of Particle Physics is not complete:

The usual quantum field theoretical formulation of gravity does not work, due to incurable divergences



$$\frac{1}{m_{pl}^4}\int^{\infty} dE'\,(E')^3 \;\to\; \infty$$

- There are conceptual difficulties with quantum black holes...

Expressions of a deepty-rooted problem:

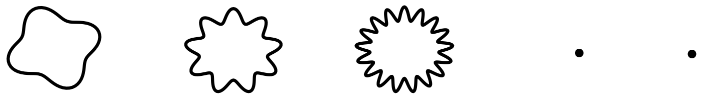
Apparent incompatibility of Quantum Mechanics and general Relativity

➡ New concepts are necessary....like string theory

As a "by-product", it also provides the **grand unification** of all particles and their interactions !

String-Theory

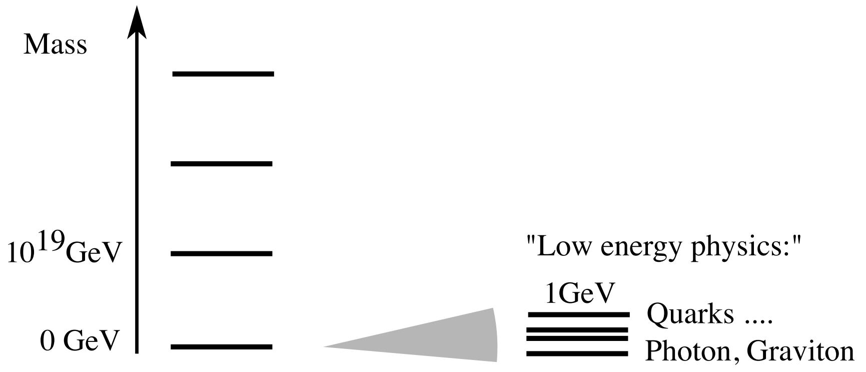
- small (10^{-34}m), one-dimensional objects:



scale determined by gravitational coupling:

$$G_N = \frac{1}{{m_{\text{planck}}}^2}\,, \qquad m_{\text{planck}} \sim 10^{19}\text{GeV}$$

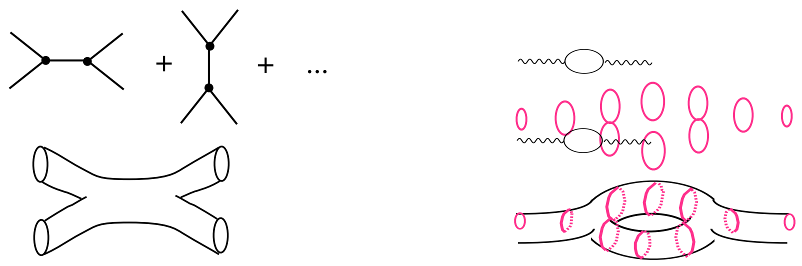
- Excitation spectrum:



"elementary" Particles of the Standard Model = (almost) massless zero modes

String Theory as 2d Field Theory

- Strings trace out two-dimensional "world-sheets" Σ_g :



- Perturbative string theory = 2d field theory on Riemann surfaces
  Building blocks: can be eg. free 2d bosons $X^a,\ X_\mu$
  $(X_\mu(z) : \Sigma_g \to \mathbb{R}^D)$

Variety of field operators in D-dimensional space time ↔ Simple combinatorics of 2-d field operators

graviton	$g_{\mu\nu}$	$=$	$\bar\partial X_\mu(\bar z)\,\partial X_\nu(z)$
gauge field	A^a_μ	$=$	$\bar\partial X_\mu(\bar z)\,\partial X^a(z)$
Higgs boson	Φ^{ab}	$=$	$\bar\partial X^a(\bar z)\,\partial X^b(z)$

Intrinsic unification of particles + interactions !
In particular, gravity is automatically built in.

The String "Miracle"

- **Perturbative effective action** in D-dimensional space-time:

2d action (eg. free)

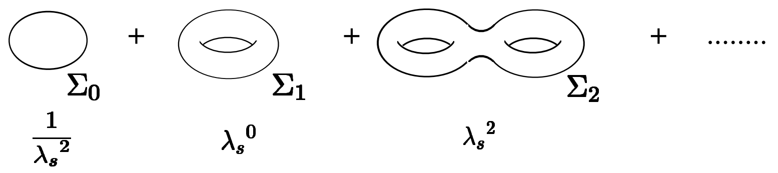
$$S_{\text{eff}}(g_{\mu\nu}, A_\mu, ..) = \sum_{\Sigma_i} e^{-\phi\chi(\Sigma_i)} \int_{M(\Sigma_i)} \int d\psi dX..\ e^{\overset{\downarrow}{\int d^2z\,\mathcal{L}_{2d}(\psi, X, .., g_{\mu\nu}, A_\mu, ..)}}$$

$$= \int d^Dx\,\sqrt{-g}\,[\underset{\uparrow}{R} + \text{Tr}F_{\mu\nu}F^{\mu\nu} + \ldots] + \underset{\uparrow}{\mathcal{O}}({m_{\text{planck}}}^{-1})$$

general relativity, gauge theory etc

small string corrections
$m_{\text{planck}} \sim 10^{19}\text{GeV}$
infinitely many predictions

``Loop expansion" = sum over 2d topologies, weighted by $e^{-<\phi>\chi} = {\lambda_s}^{-\chi}$



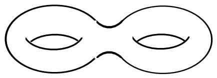
Only one (UV finite) "diagram" at any given order in perturbation theory

- **Discrete reparametriztions** of Σ_i have no analog in particle theory; important to make sense eg. of graviton scattering.
  ``Feynman rules" are substantially different from particle theory.

  String theory, even in perturbation theory, is **more** than just having infinitely many particles plus a cutoff !

Consistency Conditions

The 2d theory cannot be arbitrary but is highly constrained

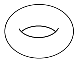


2d properties ⟶ D-dimensional properties

- **Conformal invariance** ⟶ implies gauge and general coordinate invariance in spacetime

  consistency requires D=26

- **Modular invariance** ⟶ strongly constrains spectrum



absence of UV divergences and anomalies

consistency allows only few possible spectra

- **2d Supersymmetry** ⟶ space-time fermions

  does **not** imply that D-dimensional

  consistency requires D=10

Conformal Field Theory

Is a tool set by which one can relatively easily do explicit computations, in terms of simple building blocks

- Stress-Energy Tensor T(z) = generator of conformal transformations

$$z \to f(z)$$

... satisfies an operator product algebra:

$$T(z)\cdot T(0) \sim \frac{c}{2}\frac{1}{z^4} + \frac{2}{z^2}T(0) + \frac{1}{z}\partial T(0)$$

"**central charge c**" is an anomaly, an obstruction to

For D free 2d scalars: $T_X(z) = \sum : \partial X^\mu \partial X_\mu : (z)$

each contributes 1, so

$$\boxed{c_X = D}$$

- For theories with more symmetries, like supersymmetry or gauge symmetries, one has a corresponding generalization of this current algebra

The Bosonic String

- Polyakov-action:

$$S_X = \frac{1}{4\pi\alpha'} \int d^2z \sqrt{|g|}\, g^{ab} \partial_a X^\mu \partial_b X_\mu \qquad \alpha' \sim (m_{\text{planck}})^{-2}$$

reduces in conformal gauge (2d metric g->1)
to a CFT of D free scalar fields $X_\mu(z,\bar{z}) : \Sigma_g \to \mathbb{R}^D$

- Gauge-fixing of 2d reparametrization symmetries:

$$S_{gh} = \frac{1}{2\pi} \int d^2z\, b\partial c$$

Ghost stress-tensor $T_{gh}(z) = (\partial b)c - 2\partial(bc)$ has central charge

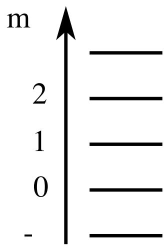
$$c_{gh} = -26$$

- Consistent quantization requires total central charge to vanish:

$$c_{tot} \equiv c_X + c_{gh} = D - 26 = 0$$

⇒ Bosonic string exists only in 26 dimensions !

- Quantization also implies a shift of vacuum energy:



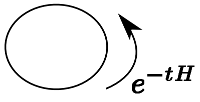
massive higher spin fields

massless Graviton, Dilaton, Tensorfield

Tachyon ⇒ No stable vacuum !

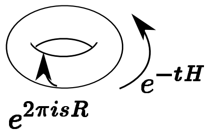
Consistency at 1-loop level

- Particles:



$$Z(t) = \text{Tr}[e^{-tH}] \quad \text{(QFT partition function)}$$

- Strings:



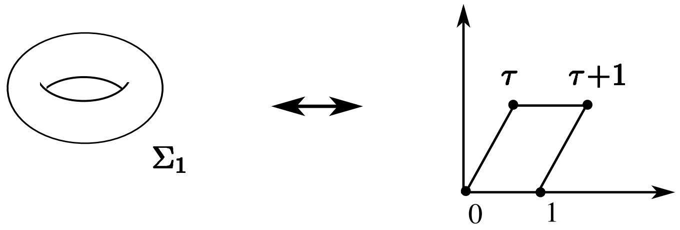
$$Z(t,s) = \text{Tr}[e^{-tH} e^{2\pi isR}]$$
$$= \text{Tr}[q^{L_0} \bar{q}^{\bar{L}_0}]$$

$$q \equiv e^{2\pi i\tau}, \qquad \tau = s + i/2\pi\, t$$

modular parameter of torus = complexified proper time

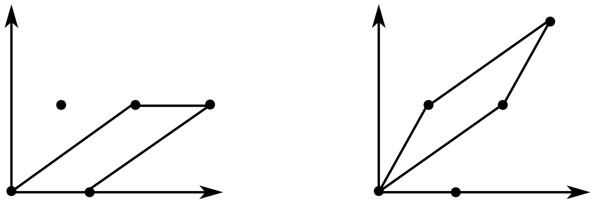
Modular transformations

- Torus is defined by identifying sides of parallelogram:



The "modular" parameter τ determines its shape

- Global reparametrizations:



$\tau \to \tau+1$ $\qquad$ $\tau \to \dfrac{\tau}{\tau+1}$

... yield **equivalent** tori !

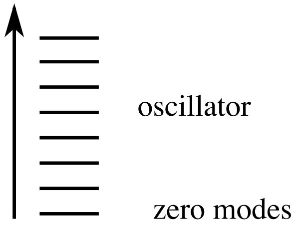
... generate the modular group, PSL(2,Z):

$$\tau \to \frac{a\tau+b}{c\tau+d}\,, \qquad a,b,c,d,\in \mathbf{Z} \qquad ad-bc=1$$

- Physical amplitudes must be **invariant** under such modular transformations !

Modular Invariance of Partition Function

For bosonic string:



$$Z(q,\bar q) \;=\; \left(\sqrt{\mathrm{Im}\tau}\,\eta(q)\eta(\bar q)\right)^{2-D}$$

where $\eta(q) \;=\; q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$ (Dedekind function)

is the (inverse) oscillator partition function of a scalar field

It has well-defined modular properties, eg: $\eta(\tau+1) \;=\; e^{i\pi/12}\eta(\tau)$

The vacuum amplitude is indeed modular invariant.

- This "global consistency condition" has no analog in particle QFT; and is responsible for many stringy features...

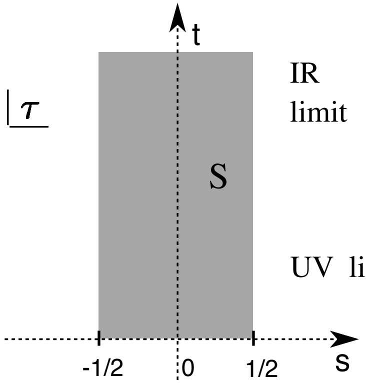
Vacuum amplitude at 1-loop

- To obtain vacuum amplitude, still need to integrate; try:

$$\mathcal{A} \;=\; \int_{-1/2}^{1/2} ds \int_0^{\infty} \frac{dt}{t^2}\, Z(s,t) \;=\; \int_{\substack{\text{strip}\\ S}} \frac{d^2\tau}{\text{Im}\tau^2}\, Z(q,\bar{q})$$

projection on R=0

t = proper time
$t \to \infty$



IR
limit

UV limit: divergence from

$$\frac{dt}{t^2}\;,\quad t \to 0$$

However, this is all-too-naive particle field theory thinking.....

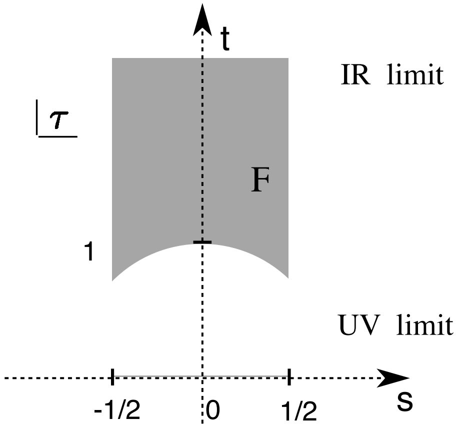
in string theory,

$$\tau \;=\; s + i/2\pi\, t$$

has a definite geometrical meaning,
namely it is the modular parameter of a torus !

UV and IR Divergences

- In string theory, we need to integrate precisely **once** over **all inequivalent shapes** of the torus.
These are described by the "fundamental region F":



IR limit

UV limit

Every point outside F is equivalent (via a modular symmetry transformation) to a point inside F**;** integrating over it would be overcounting

As t=0 is not integrated over, there is no UV divergence possible

(NB: is more than a cutoff...)

- However, there is an IR divergence for large t

$$Z(q,\bar{q}) \;\sim\; \frac{1}{q\bar{q}} + 24\left(\frac{1}{q}+\frac{1}{\bar{q}}\right) + 576 + \ldots...$$

Pole is due to Tachyon state (vacuum instability)

→ Superstrings !

Superstrings

- 2d supersymmetry $\{X_\mu, \psi_\mu\}$ --> **space-time** fermions $\Psi_\alpha \sim \sqrt{\psi_\mu}$
  typically, but not necessarily supersymmetric in space-time !

- Typically have no tachyons -- only known consistent theories...

- Two formulations: "Green-Schwarz": covariant, but difficult to quantize

  "Neveu-Schwarz-Ramond": easier to quantize, but not manifestly covariant

- From 2d NSR perspective: CFT -> SuperCFT

$$c_{tot} \;=\; \underset{X_\mu}{D} + \underset{\psi_\mu}{\tfrac{1}{2}D} \underset{b,c}{- 26} + \underset{\beta,\gamma}{11} \;=\; 0$$

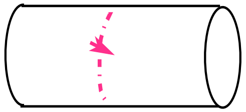
→ Critical dimension for superstrings is D=10

- Main novel technical ingredient:

  boundary conditions of the 2d fermions

Spin Structures

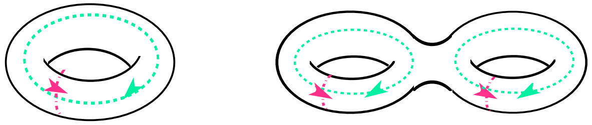
- 2d fermions can have non-trivial boundary conditions:



$$\Psi(e^{2\pi i}z) \;=\; \pm\Psi(z) \quad \begin{cases} + & \text{“Neveu-Schwarz”} \\ - & \text{“Ramond”} \end{cases}$$

The **R-sector** leads to space-time **fermions**, the **NS-sector** to **bosons** !

Each cycle can be either periodic (P) or anti-periodic (A)



- The partition function of a fermion depends on the "spin structure":

$$\begin{aligned} Z_{PP}(q) &= \mathop{\mathrm{Tr}}_{R}[(-1)^F q^{L_0}] \;=\; 0 \\ Z_{PA}(q) &= \mathop{\mathrm{Tr}}_{R}[q^{L_0}] \;=\; \prod(1+q^n) \\ Z_{AP}(q) &= \mathop{\mathrm{Tr}}_{NS}[(-1)^F q^{L_0}] \;=\; \prod(1-q^{n-1/2}) \\ Z_{AA}(q) &= \mathop{\mathrm{Tr}}_{NS}[q^{L_0}] \;=\; \prod(1+q^{n-1/2}) \end{aligned}$$

These "theta"-functions have well-defined modular properties .

- Modular invariance of the partition function requires particular, consistent choices for the boundary conditions, and these determine the physical spectrum

→ Modular invariance strongly constrains the possible physical spectra

.... chiral anomalies always cancel

Modular Invariant Partition Functions

- How to construct modular invariant sums over spin structures ?

$$\sum_{\substack{\text{spin}\\ \text{structures}}} \int [dX\, d\Psi]\, e^{-S[X,\Psi]} \;=\; \operatorname*{Tr}_{\substack{\text{spin}\\ \text{structures}}} \left[e^{-tH} e^{2\pi i s R}\right]$$

Systematic procedure: map this to certain lattice sums !
Modular invariance $\longleftrightarrow$ **self-duality of lattice**,
easy classification of all possibilities

- The result is various possibilities in D=10 :

1) Holomorphic and anti-holomorphic sectors separately invariant

$$Z^{IIA}(q,\bar{q}) \;=\; Z(\bar{q})^{+}\, Z(q)^{-} \qquad \text{"Type IIA"}$$

$$Z^{IIB}(q,\bar{q}) \;=\; Z(\bar{q})^{+}\, Z(q)^{+} \qquad \text{"Type IIB"}$$

$$Z^{\text{het}}(q,\bar{q}) \;=\; Z_{\text{bos.}}(\bar{q})\, Z(q)^{+}$$

"heterotic"
(two kinds, related to
uniqueness of self-dual lattices:

2) Non-trivial correlation of spin structures

This gives various non-supersymmetric theories,
only one of which is tachyon-free

- In addition, there is one more "open" supersymmetric string

Supersymmetric String Theories in D=10

- By combining superstring (S) and bosonic string (B) building blocks, one can construct five types of string theories in D=10:

Combination	Name	Gauge group
$S \otimes \bar{S}^{\dagger}$	Type IIA	$U(1)$
$S \otimes \bar{S}$	Type IIB	$-$
$S \otimes \bar{B}$	Heterotic	$E_8 \times E_8$
$S \otimes \bar{B}'$	Heterotic$'$	$SO(32)$
$(S \otimes \bar{S})/Z_2$	Type I (open)	$SO(32)$

- These theories have one dimensionful parameter, the string tension $\alpha' \sim (m_{\text{planck}})^{-2}$, besides the coupling $\lambda_s = e^{<\Phi>}$.

- They have very different spectra (c.f., gauge groups) !

String Theories in Lower Dimensions

W. Lerche, ISTAPP 2011
Istanbul Part 2

Recall:
Perturbative constructions (based on 2d conformal field theory on Riemann surfaces) , subject to certain consistency requirements, lead to

- 5 supersymmetric consistent string theories in D=10:

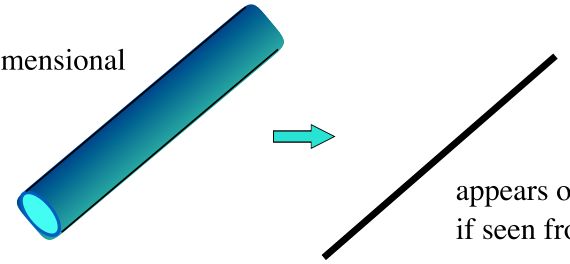
Combination	Name	Gauge group
$S \otimes \bar{S}^\dagger$	Type IIA	$U(1)$
$S \otimes \bar{S}$	Type IIB	–
$S \otimes \bar{B}$	Heterotic	$E_8 \times E_8$
$S \otimes \bar{B}'$	Heterotic′	$SO(32)$
$(S \otimes \bar{S})/Z_2$	Type I (open)	$SO(32)$

- Spectra are highly restricted by anomaly cancellations (guaranteed by modular invariance)
- They have very different perturbative spectra in 10d; Naively, all reason to believe that they are different theories... !
- But we don't live in D=10 but in D=4........

"Compactification" of Dimensions

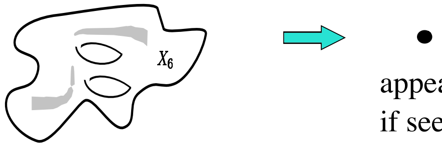
Rolling up:

three-dimensional



appears one-dimensional,
if seen from a distance !

compact six-dimensional
manifold



appears zero-dimensional,
if seen from a distance !

- assume space-time has form:

$$\mathbb{R}^{10} \longrightarrow \mathbb{R}^4 \otimes X_6$$

where X_6 is some 6-dim manifold of very small size

10-dim

$$A_M \longrightarrow \{A_\mu, \phi_i\}$$
$$g_{MN} \longrightarrow \{g_{\mu\nu}, A_{\mu\, i}, \phi_{i\,j}\}$$

4-dim fields

At large enough distances, or low energies, the extra dimensions are hidden and the theory effectively looks four-dimensional !

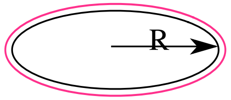
Toy Model: Compactification on Circle S^1

Hamiltonian:

$$H = \frac{m^2}{R^2} + n^2 R^2 - 1\ , \qquad m, n \in \mathbf{Z}$$

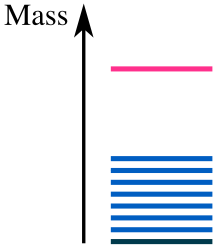
discrete momentum states
(like in particle QFT,
"Kaluza-Klein"
spherical harmonics)

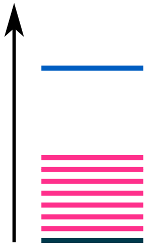
winding states
(specifically stringy)



become light as $R \to \infty$

become light as $R \to 0$





Important: theory is invariant under exchange

$$R \leftrightarrow \frac{1}{R}\ , \qquad m \leftrightarrow n$$

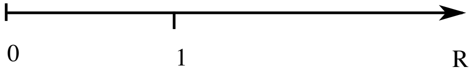
"T-Duality"

Large-Small Radius "T"-Duality

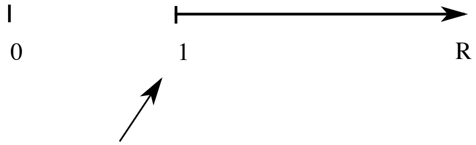
No **physical distinction** between large or small compactification radius !

Compare parameter ("**moduli**") spaces of inequivalent vacua:

- Particle QFT on circle with radius R:



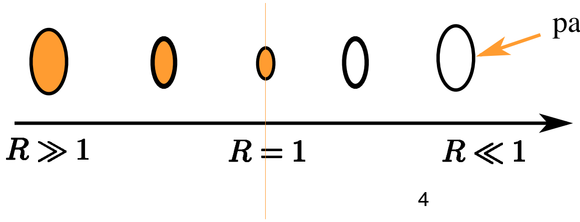
- String theory on circle with radius R:



effective minimal length scale

String theory defines a novel kind of geometry: **"Stringy Geometry"**

(here: geometrically distinct circles
become identified in string theory)

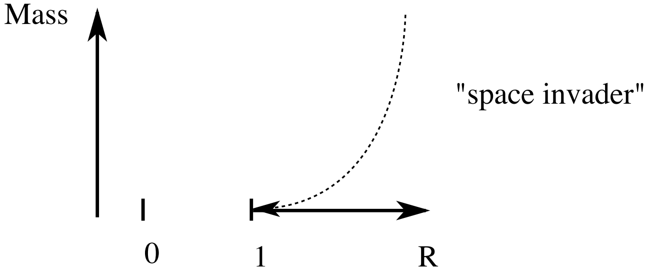


particle physics singularity

Extra Gauge Symmetry

- Typically, interesting phenomena arise at special (boundary) points of parameter space:

Consider the mass of momentum-winding states with $(m, n) = (1, \pm 1)$



extra massless states at self-dual radius R=1
.... SU(2)xSU(2) gauge fields

changing the radius away from R=1, gives these gauge fields
a non-zero mass

→ Stringy Higgs effect

- General principles:

compactification induces additional states and geometrical parameters

these correspond to undetermined field theory VEV's $R \sim \langle \Phi \rangle$

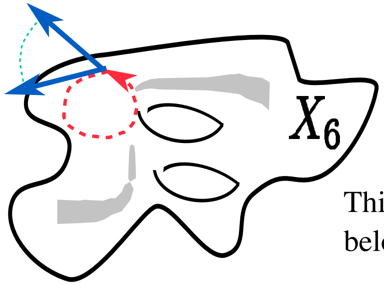
fixed points of duality transformations: extra gauge symmetries

---

Holonomy and Supersymmetry

N=1 Supersymmetry is phenomenologically desirable, and technically required for having a tractable theory with a stable ground state.

Consider looping a tangent vector on the 6-dimensional compactification manifold:



This generically induces a rotation which belongs to the "**holonomy**" group SO(6)

Condition for supersymmetry:
Existence of a covariantly constant spinor

A priori, spinors on some X6 transform as the 4-dimensional spin representation of SO(6).
Assume a complex "Kahler" manifold with holonomy group $\mathcal{H}(X_6) \simeq SU(3)$
and thus $\mathbf{4} \to \mathbf{3} \oplus \mathbf{1}$

The singlet component is supposedly covariantly constant and represents the unbroken supercharge:

$$\nabla \Psi = 0 \longrightarrow \gamma^k R_{ik} \Psi = 0$$

Represents definition of a "**Calabi-Yau**" manifold:

"complex Kahler manifold with vanishing first Chern class" $c_1 = R_{ik} = 0$

Calabi-Yau Compactifications

To ensure supersymmetry in D dimensions, X must be a multi-torus, or a **Calabi-Yau**-manifold with holonomy

$$\mathcal{H} \subset SU(5 - D/2)$$

Possibilities for having N supersymmetries in various dimensions:

D	X_{10-D}	$\mathcal{H}$	Type II string on X_{10-D}	heterotic string on X_{10-D}
8	T_2	1	$N = 2$	$N = 1$
6	T_4	1	$N = 4$	$N = 2$
	$K3$	$SU(2)$	$N = 2$	$N = 1$
4	T_6	1	$N = 8$	$N = 4$
	$K3 \times T_2$	$SU(2)$	$N = 4$	$N = 2$
	Calabi-Yau	$SU(3)$	$N = 2$	$N = 1$

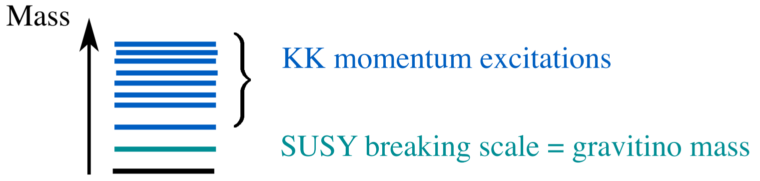
Phenomenologically most interesting
(can have chiral fermions)

(Un)fortunately, plenty of possibilities....

Supersymmetry Breaking

in perturbative heterotic string compactifications

- As a dogma, one likes approximate supersymmetry, spontaneously broken only at the TeV scale in order to protect the weak scale from renormalization
- Generic string prediction:
Modular invariance implies that SUSY breaking scale is scale of compact dimensions !



10d parameters: string length $1/\ell_s = m_s$

string coupling $\lambda_s = e^{<\Phi>}$

4d parameters: Planck scale $m_{\rm planck} \cong 10^{19}$GeV

gauge $1/g^2 \cong 1/25$

Compare:

$$m_s = g\, m_{\rm planck}$$

$$\lambda_s = g\frac{\sqrt{{\rm Vol}(X_6)}}{{\ell_s}^3}$$

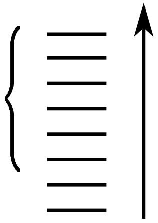
string coupling grows for largeX_6 !

If we have 1/TeV-sized compact dimensions, heterotic strings must be strongly coupled !

Perturbative description breaks down... need to use non-perturbative dualities....

Predictivity

- Is there more than those generic predictions of string theory ?



In principle: infinitely many predictions !
(spectrum very tightly constrained

In practice: almost no predictions in zero mode sector ⇢

Properties of massless sector

⟷

Properties of compactification space

= **choice of vacuum state** $R \sim \langle \Phi \rangle$

.. not much determined by 10D string theories !

Analogous to spontaneously chosen direction of magnetization in a ferro-magnet, which is also not determined by fundamental principles....

The specific properties of the standard model may not have any particular reason at all ...
they simply might be "frozen historical accidents"

Doing away with Supersymmetry ?

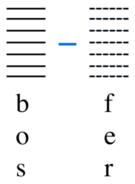
- Supersymmetry is **not** an intrinsic prediction of string theory !
  It has been invented to remedy renormalization properties

  However, string theory is more clever than QFT, and naive particle physics intuition can be very misleading.....

- Consider eg vanishing of 1-loop vacuum energy (cosmolog. const)

  - Particle theory:

$$\mathcal{A}_{\text{part}} = \int_t \mathrm{Tr}[e^{-tH}] = \int \sum$$



$$\overset{!}{=} 0$$

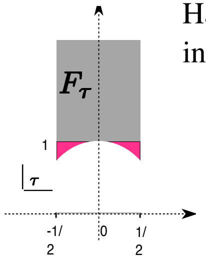
(level-by-level cancellation)

  - String theory:

$$\mathcal{A}_{\text{string}} = \int_{F_\tau} \mathrm{Tr}[e^{-tH} e^{2\pi i s R}] = \mathcal{A}_{\text{part}} + \text{stringy stuff} \overset{!}{=} 0$$

May vanish only after modular integration, without integrand being zero

Has no analog in particle theory



Amplitudes can vanish even without supersymmetry !

Topology and Zero-Modes

- Since the excitation spectrum is typically 10^19GeV, we are mainly interested in the **massless zero-modes**.

  ...these probe the global, topological properties of X_6

- Expand 10-dim field on $\mathbb{R}^4 \otimes X_6$:

$$\Phi = \sum_i \phi_i^{(4)} \omega_i^{(6)}$$

Laplace operator:

$$\Delta^{(10)} = \Delta^{(4)} + \Delta^{(6)}$$

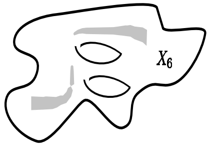
(mass term in 4d

The 4-dim fields $\phi^{(i)}$ are massless if $\omega^{(i)}$ are **harmonic differential** forms on X_6 :

$$d\omega_i^{(6)} = d^*\omega_i^{(6)} = 0$$

- Such forms play a crucial role in algebraic geometry, and indeed reflect the topology ("cohomology") of the compactification space...

Their numbers are (roughly!) given by the numbers of higher-dimensional "holes" within X_6



More precisely, the spectrum is given by the topological "**Hodge numbers**" associated with every Calabi-Yau X_6:

$$h^{pq} = \dim H^{p,q}_{\bar{\partial}}(X_6, \mathbf{C})$$

of which only h^{11} and h^{21} are independent

- For the heterotic string compactified on some Calabi-Yau manifoldX6, we typically get the an effective N=1 supergravity theory plus various extra gauge and matter ("chiral") super-fields: $\mathbf{\Phi}_i \equiv (\phi_i, \psi_i)$

- graviton and gravitino, $g_{\mu\nu}, \Psi_{\mu\alpha}$
  dilaton-axion superfield S,

- gauge bosons and gauginos corresponding to

- h^{21} matter superfields in the $\underline{27}$ of E_6,

  h^{11} matter superfields in the $\underline{27}^*$ of E_6

  h^{21} matter superfields: complex structure (shape) moduli,

  h^{11} matter superfields: Kahler (size) moduli,

  H^1(End T) matter superfields: gauge singlets,

- Net # of left- minus right-handed families = 1/2 "Euler number" :

$$\frac{1}{2}\left|h^{11}(X_6) - h^{21}(X_6)\right| \equiv \frac{1}{2}|\chi(X_6)|$$

This gives a natural **repetitive structure** of "particle generations" !

Generic Properties of D=4 Compactifications

On typical Calabi-Yau manifolds, heterotic strings provide thus effective particle field theories in D=4 with:

- Gravity
- Gauge symmetries
- Chiral fermions
- Repetitive generation structure
- Higgs mechanism
- (Supersymmetry)

... the **generic** features of what we do see in nature, all coupled together in a truly consistent manner !

This is the main achievement of string theory

But there are also quite a few unpleasant features and unsolved problems, eg:

- Vacuum state indeterminacy
- plenty of scalar fields, massless "dilaton" field
- How to get rid off supersymmetry
- Why is the cosmological constant (almost) zero

Vacuum Degeneracy, “Landscape” Problem

The lack of predictivity at low energies is the most serious challenge for the credibility of string theory !

- There are plenty of Calabi-Yau spaces, and it is not clear why any one should be singled out. Nor why D=4 would be preferred at all ....
- On top of that there are 5 theories in D=10, all of which give four-dimensional theories upon appropriate compactification.

If one is the fundamental theory,
what is the meaning of the others ?

- Each Calabi-Yau space leads in general to a different spectrum in D=4, and, even worse, has a huge **parameter space** by itself.

We don't have good answers for the vacuum degeneracy problem, but have made exciting progress in understanding the second question.... see lecture 3.

Geometrization of Coupling Constants

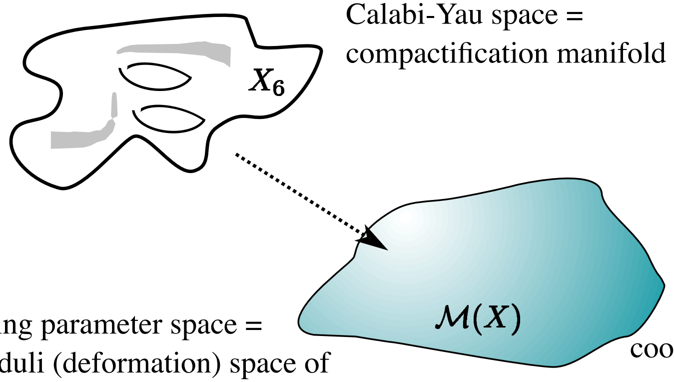
- In D=10, there is essentially just one free parameter, namely the string coupling; it corresponds to the VEV of the dilaton field:

$$\lambda_s = e^{<\Phi>}$$

- Compactification to lower dimensions makes theories much **more complex** than in D=10 !

(Recall compactification on circle: radius gives masses of gauge bosons)

The geometrical parameters ("moduli") that govern the **shape** of X_6 become free physical vacuum VEV's (like in couplings $\langle \Phi_i \rangle \psi\psi$), which are not determined by the 10d theory.



Calabi-Yau space =
compactification manifold

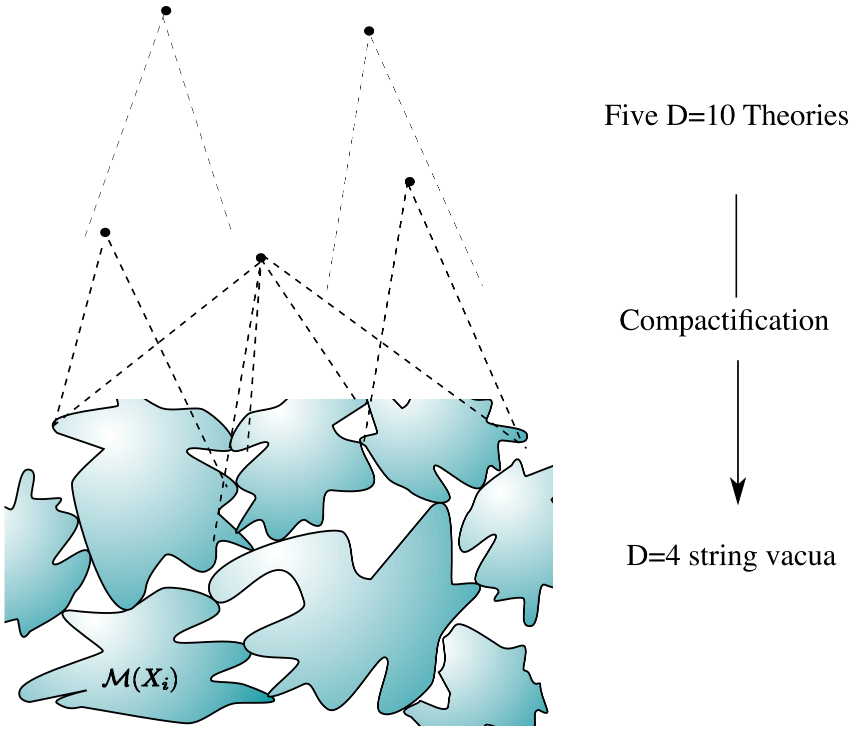
String parameter space =
moduli (deformation) space of
compactification manifold

coordinates
= moduli fields
$\langle \Phi_i \rangle$

Typically, $\dim \mathcal{M}(X_6) \;\cong\; 100$

- Almost every coupling of the effective 4 dimensional theory has a geometric interpretation rooted in the properties of X_6

N=2 SUSY String Compactifications in D=4



Five D=10 Theories

Compactification

D=4 string vacua

Each of the ~ 10^8 blobs corresponds to a continuous, ~100-parameter family of string theories in D=4

Consistency is restrictive primarily only for the high-dimensional string theories !

SUSY Effective Actions in d=4

- A computation of the general full string effective action is not feasible!

  However, in SUSY theories we can go pretty far:
  they are partially characterized by **holomorphic functions** $f(\phi)$
  of the massless moduli (scalar) fields.

  These are protected by non-renormalization theorems, and largely determined given by topological properties of X_6.

- Examples for such holomorphic functions:

$$N=4: \quad \text{gauge coupling } \tau(\phi), \text{ higher derivative terms}$$
$$N=2: \quad \text{gauge coupling } \tau(\phi) = {\partial_\phi}^2 F(\phi), \text{ (Prepotential) } \ldots$$
$$N=1: \quad \text{Superpotential } W(\phi), \text{ gauge coupling } \tau(\phi), \ldots$$

- E.g., superpotential for the fields ϕ_a in the 27 of E_6

$$W(\phi) \;=\; \phi_a \phi_b \phi_c \int_{X_6} \omega_a^{1,1} \wedge \omega_b^{1,1} \wedge \omega_c^{1,1} \;+\; \text{corr.}$$

intersection #'s

non-perturbative
instantion corrections -
how to compute ?

World-Sheet Instanton Corrections

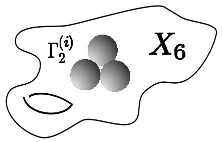
- Despite non-perturbative corrections, certain (holomorphic) quantities like superpotentials can often be computed exactly

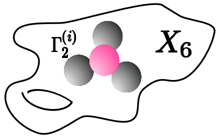
- eg gauge couplings in N=2 SUSY (type II strings on CY) depend on moduli fields t:

$$\tau_{\text{eff}}(t) \;\equiv\; \frac{1}{2\pi}\theta_{\text{eff}}(t) + 2\pi i \frac{1}{{g_{\text{eff}}}^2(t)} \;=\; \tau_0 + \sum_{\ell=1}^{\infty} c_\ell \log[1 - e^{2\pi i \ell t}]$$

bare coupling =
topological intersection number

Instanton corrections:
string world-sheets wrapping
around 2-cycles $\Gamma_2^{(i)}$





Nonperturbative in 2d,
but tree-level in 4d

- How to determine the unknown coefficients c_ℓ ?

  Mathematically, this corresponds to summing up all maps from the string world-sheet into the Calabi-Yau:

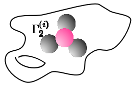
$$S^2 \longrightarrow X_6 \qquad\qquad e^{-S_{\text{inst}}} = e^{2\pi i t}$$

..which is an extremely hard problem in algebraic geometry!

Mirror Symmetry

- Is a first example of a very non-trivial duality, in fact it is a generalization of (perturbative) T-duality

type IIA string on CY ⟷ type IIB string on CY'



complicated world-sheet instantion corrections; 2-cycles Γ_2

difficult

$$\tau_{\rm eff} = \sum_{\ell=1}^{\infty} c_\ell \log[1 - e^{2\pi i \ell t}]$$

(CY' = a quite different "mirror manifold")

classical tree-level intersection geometry of 3-cycles Γ_3

easy

boils down to computing certain integrals:

$$\tau_{\rm eff}(t) = \partial_\tau \int_{\Gamma_3} \Omega^{(CY')}(t)$$

which determines all c_ℓ .

..and the mathematicians

By mapping a **complicated** problem to a **simple** one, via duality, one can obtain highly non-trivial, exact results !

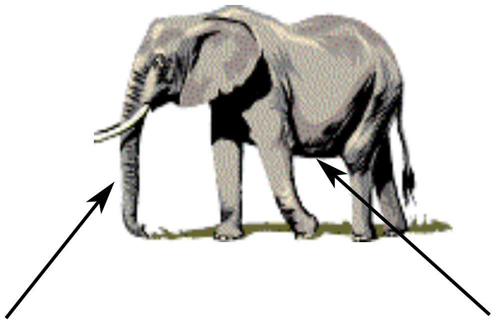
.. and as we will see, similar methods work at the non-perturbative level as well !

Non-Perturbative String Physics

W. Lerche, ISTAPP 2011
Istanbul Part 3

- We have seen that there are 5 superstring theories in D=10, leading to very many different D=4 compactifications

- But it turns out that thinking in terms of perturbation theory only, we are effectively blindfolded...



Approximate description in terms of cylinder geometry

Description in terms of cylinder geometry is not useful here

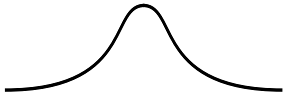
(And yes, I had the elephant before Brian Greene...)

Non-perturbative Equivalences

- Map **solitonic** (non-perturbative, non-local extended) degrees of freedom to **elementary** (perturbative, local) ones, and vice versa

"local" "non-local"

• $\Phi(x)$



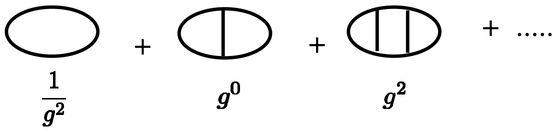
elementary field ⟷ collective excitation ("Soliton")

These are simply two ways to describe one and the same physical degree of freedom

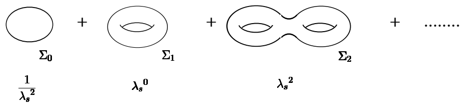
Simplest example is 2d Ising model:

fermion $\Psi \longleftrightarrow :e^{i\Phi}:$ soliton

- Intrinsically a quantum phenomenon !
- Duality typically **maps weak** to **strong coupling**: $g \to \frac{1}{g}$



string theory:



Therefore it cannot be captured in perturbation theory !

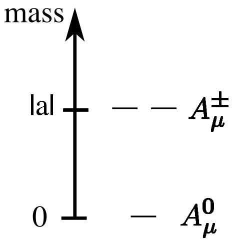
Usual QFT, Lagrange formalisms fail and must be **abandoned** ..

Montonen-Olive Duality

- Cons. SU(2) gauge theory with Higgs field in the adjoint representation

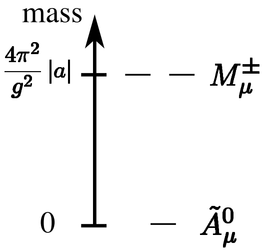
For non-zero Higgs VEV: $a \equiv \langle\phi\rangle$ the symmetry is broken to U(1), and the charged gauge fields get mass due to the Higgs mechanism

Perturbative spectrum:



elementary gauge fields

Non-perturbative spectrum:



solitonic magnetic monopoles

- Duality transformation:

$$\begin{aligned} F_{\mu\nu} &\to \tilde{F}_{\mu\nu} \qquad (\vec{E} \to \vec{B},\ \vec{B} \to -\vec{E}) \\ q &\to m \ \ (\text{charges}\ ) \\ a &\to a_D = \frac{4\pi}{g^2}a \ \ (\text{Higgs VEVs}) \\ \frac{4\pi}{g^2} &\to \frac{g^2}{4\pi} \ \ (\text{couplings}) \end{aligned}$$

In the dual theory, the magnetic monopoles behave like the gauge bosons in the original theory, and are massive via the dual Higgs field

Montonen-Olive:
The theory might be **invariant** under this non-perturbative transformation !

Supersymmetry and BPS-States

- Supersymmetry in itself may not be not fundamentally important, but it allows us do to non-trivial exact computations, by virtue of its **non-renormalization properties** that **protect** many quantities from perturbative corrections.

- In particular, quantities related to "BPS"-states:

$$Q_\alpha \big|\mathrm{BPS}\big\rangle \;=\; 0$$

From the algebra of supersymmetry charges

$$\{Q_\alpha, Q_\beta\} \;=\; \gamma^\mu_{\alpha\beta} P_\mu + \delta_{\alpha\beta} Z$$

("central charge" Z can be eg. U(1) charge)

follows for such BPS-states that their mass is **exactly** given

$$\boxed{m^2 \;=\; |Z|^2}$$

- Idea: Find that in semi-classical approximation some state is BPS - this implies it has less degress of freedom than a generic state ("short SUSY multiplet")

But under smooth perturbative and non-perturbative corrections, the number of degrees of freedom cannot jump

➡ The state is BPS also in the full quantum theory, and in particular its mass is **exactly** known !

The BPS property is the quintessential basis of our modern non-perturbative techniques.

S-Duality in N=4 SUSY Gauge Theory

- N=4 SUSY: no quantum corrections to gauge coupling, and BPS masses:

Montonen-Olive duality is indeed exact !

- But there is more structure: include the theta-angle to define a complexified gauge coupling,

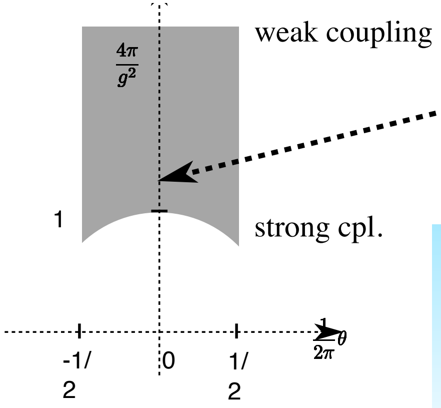
$$\tau \;\equiv\; \frac{1}{2\pi}\theta + 2\pi i \frac{1}{g^2}$$

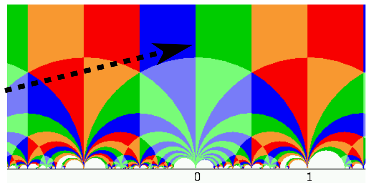
This combines the MO-duality and then theta-shift symmetry:

$$\tau \;\longrightarrow\; -\frac{1}{\tau}\,, \qquad\qquad \tau \;\longrightarrow\; \tau+1$$

These transformations generate the non-abelian, discrete "**S-duality**" group, SL(2,Z) !

- This non-perturbative symmetry group implies a non-trivial phase structure, governed by the fundamental domain:





We have seen this before: it is the fundamental region of a torus, whose group of discrete parametrizations is nothing but SL(2,Z) .....

What's the significance of this fact ???

Duality in N=2 SUSY Gauge Theories

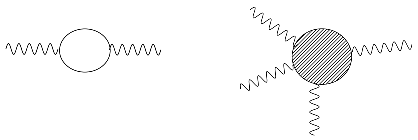
- In N=2 SYM theory, the monople masses do get renormalized, however both the gauge fields (elementary) and the magnetic monopoles (solitonic, non-local) are still BPS.

- Effective gauge coupling gets renormalized, and dependent on the Higgs field:

$$\tau(\phi) \equiv \frac{1}{2\pi}\theta(\phi) + 2\pi i \frac{1}{g^2(\phi)}$$

One knows beforehand the general form of the quantum corrections:

$$\tau(\phi) = \underbrace{\frac{2\pi i}{{g_0}^2}}_{\text{bare coupling}} + \underbrace{\frac{i}{\pi}\log\left[\frac{\phi}{\Lambda^2}\right]}_{\text{one-loop}} - \underbrace{\frac{i}{\pi}\sum_{\ell=1} c_\ell\left(\frac{\Lambda}{\phi}\right)}_{\text{instanton correct}}$$

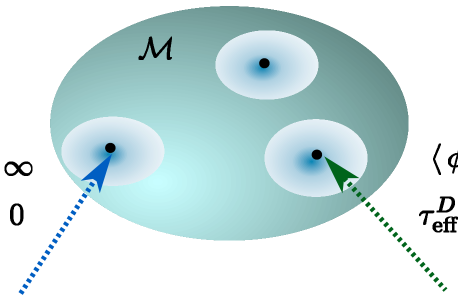


- How to determine the instanton coefficients c_1 ?

Seiberg-Witten 1994: found a surprising solution, starting from the topology of the parameter space !

Quantum Moduli Space of N=2 Gauge Theory

- Moduli (parameter) space $\mathcal{M}$: VEV of complex Higgs field ϕ (or dual ϕ_D)



$\langle \phi \rangle \to \infty$

$\tau_{\text{eff}}(\phi) \to 0$

$\langle \phi_D \rangle \to 0$

$\tau^D_{\text{eff}}(\phi_D) \equiv \frac{-1}{\tau_{\text{eff}}(\phi)} \to 0$

gauge fields weakly coupled; monopoles strongly coupled

$$\tau_{\text{eff}}(\phi) = \frac{1}{\pi}\log\left[\frac{\phi^2}{\Lambda^2}\right] - \frac{1}{\pi}\sum_{\ell=1}^{\infty} c_\ell\left(\frac{\Lambda}{\phi}\right)^{4\ell}$$

1-loop

instanton corr

SU(2) gauge theory with

gauge fields strongly coupled; massless monopoles weakly coupled, effectively look like electrons;

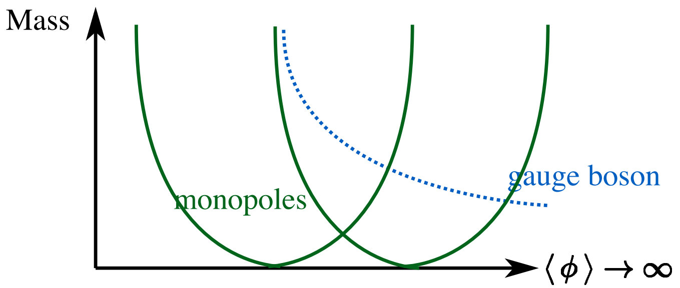
$$\tau^D_{\text{eff}}(\phi) = \frac{-1}{2\pi}\log\left[\frac{\phi_D^2}{\Lambda^2}\right] - \frac{1}{\pi}\sum_{\ell=1}^{\infty} c^D_\ell\left(\frac{\phi_D}{\Lambda}\right)^{2\ell}$$

1-loop

non-pert corr

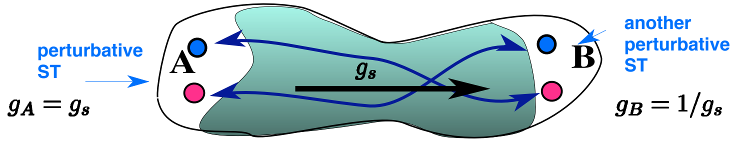
U(1) gauge theory with

Resummation of non-perturbative corrections

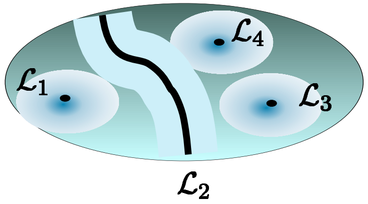


A general lesson we can abstract from this:

- In general, there is no global description that would be valid throughout the whole moduli space; **no particular lagrangian is more fundamental** than the other ones.



- Lagrangian description makes sense only in "**local coordinate patches**" covering the parameter space $\mathcal{M}$:



- These describe **different local approximations** of the same theory **in terms of different weakly coupled physical degrees of freedom** (eg, electrons or monopoles)

- The perturbative physics (local QFT) looks completely different in the various local patches (eg, different gauge groups)

➡ Concept of "fundamental degrees of freedom" is questionable,
at least

Solitons in String Theory

- What are the **analogs** of **magnetic monopoles** in string theory ?
Depending on the string model, there are various p-dimensional solitonic "**p-branes**"
(p=0: particle, p=1: string, p=2: membrane,.....),
which are not visible in perturbation theory.

- Recall gauge theory EM duality in D=4:
1-form gauge field: $A_\mu \equiv A^{(1)}$

$$\begin{tikzcd} \text{gauge} \quad A^{(1)} \arrow[r] & F^{(2)} \arrow[r, leftrightarrow, "4"] & \tilde{F}^{(2)} \arrow[r] & \tilde{A}^{(1)} \quad \text{magn monopole} \end{tikzcd}$$

- Generalized EM duality in D dimensions:

$$\begin{tikzcd} F^{(p)} \arrow[r, leftrightarrow, "D"] & \tilde{F}^{(D-p)} \end{tikzcd}$$

➡ Check what generalized gauge fields there are (D=10 Type II strings):

2-form: $B^{(2)}$ **RR tensor fields:** $A^{(p+1)}$ $\begin{cases} p = 2k & \text{IIA} \\ p = 2k+1 & \text{IIB} \end{cases}$

1. fund.

$$\begin{tikzcd} \text{strin} \quad B^{(2)} \arrow[r] & F^{(3)} \arrow[r, leftrightarrow, "10"] & \tilde{F}^{(7)} \arrow[r] & \tilde{A}^{(6)} \quad \text{"NS" 5-brane} \end{tikzcd}$$

2. RR tensors

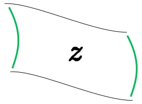
$$\begin{tikzcd} \text{"D" p-brane} \quad A^{(p+1)} \arrow[r] & F^{(p+2)} \arrow[r, leftrightarrow, "1"] & \tilde{F}^{(8-p)} \arrow[r] & \tilde{A}^{(7-p)} \quad \text{"D" (6-p)-brane} \end{tikzcd}$$

- Typically, some of those branes are BPS and we may hope to be able to do exact computations with them !

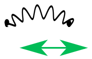
D-Branes as Dirichlet Boundary Conditions

- It was shown that sources for the RR tensor gauge fields are provided simply by Dirichlet boundary conditions for open strings

$X(z)$



$\partial_{\perp} X(z) = 0$



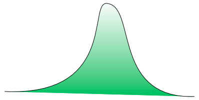
$\delta p = 0$

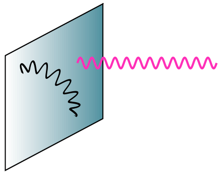
Neumann

$\partial_{||} X(z) = 0$ $\delta X = 0$

Dirichle

D-branes can thus simply be described as regions on which open strings can end.
As such, they provide a perturbative description in terms of conformal field theory.





Closed string (Type II)
p-brane soliton

↔

Open string (Type I)
elementary D-p-brane

non-perturbative

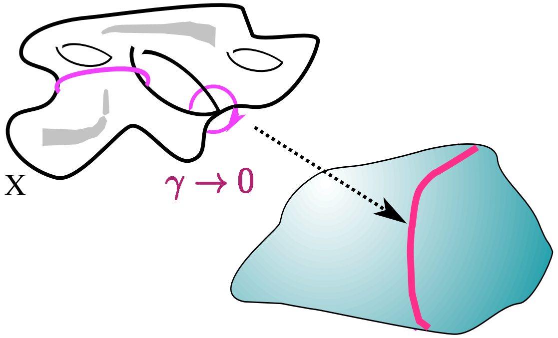
perturbative

- D-branes are thus string analogs of the elementary electrons into which the magnetic monopoles transform under S-duality.

Wrapped Branes: Non-Perturbative Extra States

- String compactification: **proliferation of physical degrees of freedom**, obtained from wrapping strings (p=1), membranes (p=2), general p-branes around non-contractible p-cycles of X

  At a given singularity in the parameter space $\mathcal{M}(X)$, a compactification manifold X becomes singular in that some p-dimensional "**vanishing cycle** γ " **shrinks to zero size**:



- This typically implies a BPS p-brane to become massless, when wrapped around γ :

$$m^2_{\text{p-brane}} = |\int_{\gamma} \Omega(X)|^2 \longrightarrow 0 \quad \text{if} \quad \gamma \longrightarrow 0$$

In an appropriate situation, the remnant of this in D=4 space-time is simply an "extra" massless particle of some kind, e.g., a Seiberg-Witten monopole or dual quark, a gauge field, quark, Higgs field....

The existence of such extra solitonic states, not seen in naive perturbative string theory, is the basis for non-trivial equivalences of string theories !

"Stringy Geometry"

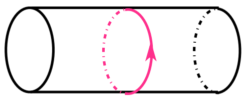
.... transcends ordinary geometry

String theory A compactified on X_A can be dual, ie, quantum equivalent, to string theory B compactified on X_B, where the manifolds X_A and X_B are completely different !

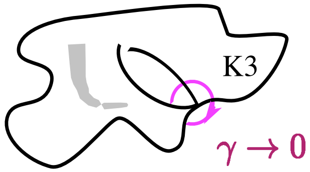
- Example in D=6: Heterotic(T_4) = Type IIA(K3) = Type I (K3)

  One and the same massless SU(2) gauge boson has the following representations in the different theories:

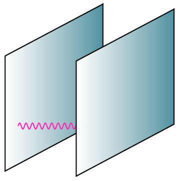
  - In the heterotic string model, as a fundamental heterotic string wrapped around a cylinder of radius R=1 (perturbative):



  - In the Type IIA string theory, as a 2-brane wrapped around a



  - In the Type I string model, as a fundamental open string stretched between D-branes, in the limit of coinciding D-branes (perturbative):

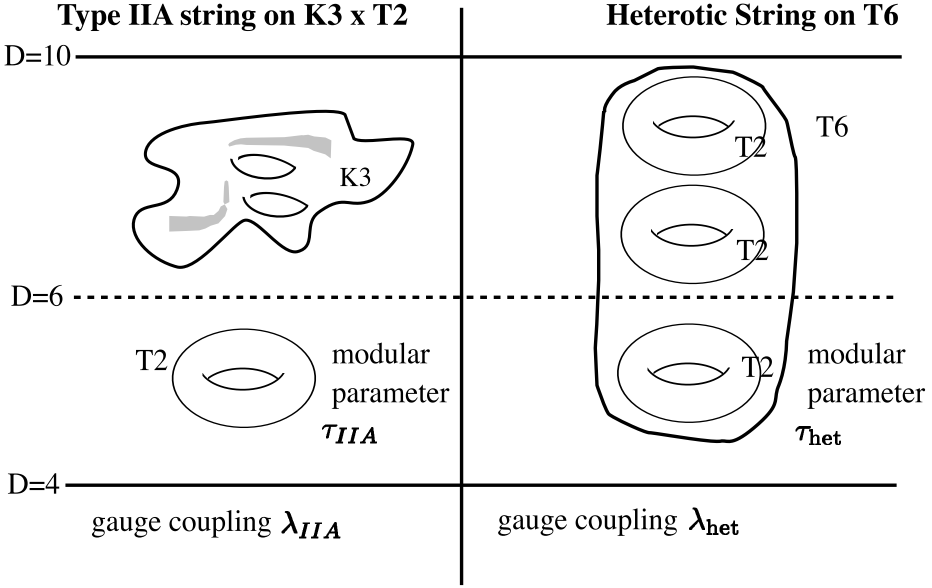


→ **These different "mathematical" geometries represent**

(here, the SU(2) Higgs model)

Geometrization of non-perturbative Dualities

- Consider duality between compactifications with N=4 SUSY in D=4:



Type IIA string on K3 x T2 | **Heterotic String on T6**

- The string duality maps geometrical compactification moduli into gauge coupling constants, and vice versa:

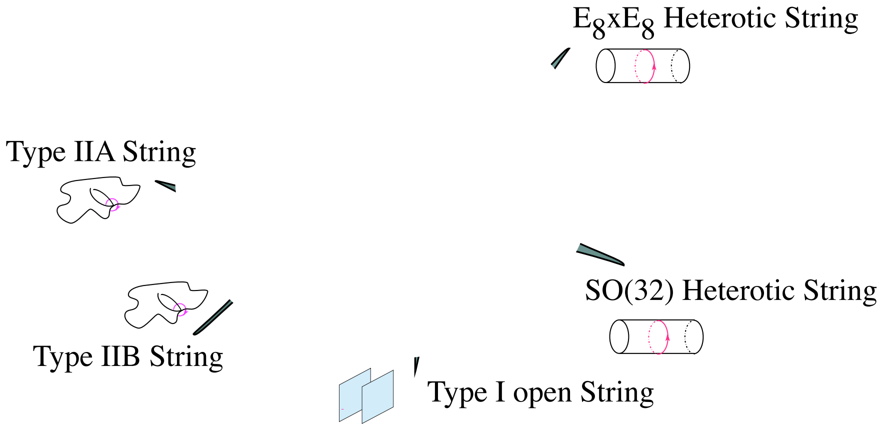
$$\tau_{IIA} = \lambda_{het}$$
$$\lambda_{IIA} = \tau_{het}$$

- In this way, the non-perturbative S-duality symmetry SL(2,Z) of the type II string is mapped to a perturbative T-duality of the heterotic string (and v.v), explaining the appearence of the non-perturbative modular geometry!

Recall that the coupling constant λ governs the perturbation expansion of quantum corrections - this means the duality maps between classical and quantum objects....
No unique distinction as to what classical and what quantum effects are !

10-D String Theories Revisited

Recall we had five string theories in D=10 -
how are they interrelated in view of the dualities ?



E_8xE_8 Heterotic String

Type IIA String

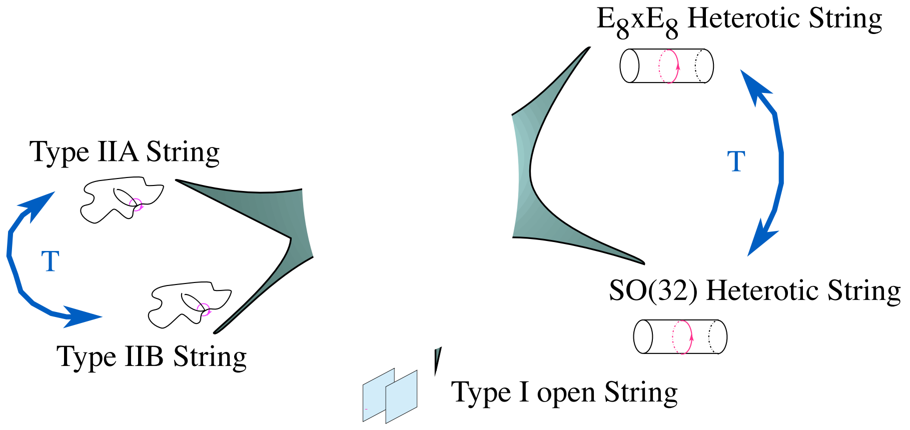
SO(32) Heterotic String

Type IIB String

Type I open String

T-Duality and Mirror Symmetry

Staying completely within perturbative CFT, we know how
to continuously deform some theories in the following way:



E_8xE_8 Heterotic String

T

Type IIA String

T

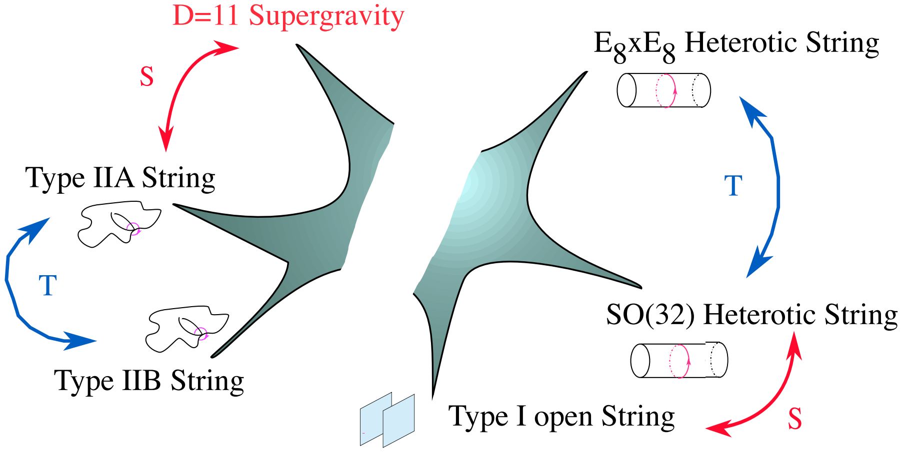
SO(32) Heterotic String

Type IIB String

Type I open String

Adding S-Duality

Staying completely within perturbative CFT, we know how to continuously deform some theories in the following way:



D=11 Supergravity

S

Type IIA String

T

Type IIB String

$E_8 x E_8$ Heterotic String

T

SO(32) Heterotic String

Type I open String

S

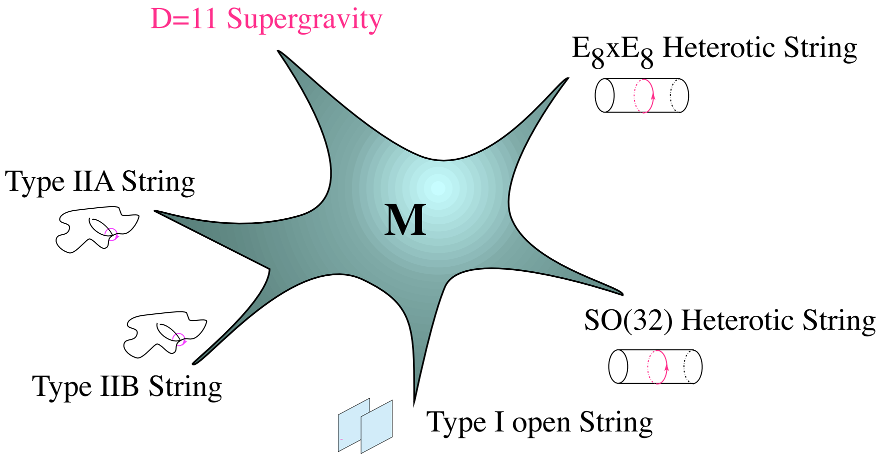
Performing strong coupling limits induces new non-perturbative relationsships !

Surprise: taking the strong coupling limit in the Type IIA string, non-perturbative states ("D0-branes") generate an **11th dimension** !

D=11 supergravity in not related to a string theory, rather is related to **supersymmetric membranes**.....

The Grand Picture

Adding certain brane backgrounds finally links all theories together:



D=11 Supergravity

$E_8 x E_8$ Heterotic String

Type IIA String

M

Type IIB String

SO(32) Heterotic String

Type I open String

All five string theories in D=10 appear as particular perturbative approximations of **one theory** !

- Just like in N=2 SYM theory, there are various parametrizations, each of which prefers certain physical excitations being as "fundamental" and weakly coupled.

Dualities take us beyond string theory ! ... M-Theory ?

"M-Theory"

- Defined to be the theory that, upon compactification on a circle, gives Type IIA string theory:

  for large R, strongly coupled Type IIA string
  for R~0, weakly coupled Type IIA string

  Fundamental degrees of freedom: "**D-particles**" (Type IIA solitons)

- Dynamics described by large-N limit of SUSY quantum mechanics:

$$\mathcal{H} \;=\; R\,\mathrm{Tr}\left\{(\partial X^i)^2 - [X^i, X^j]^2 + \Theta[\gamma X, \Theta]\right\}$$

= 10-D U(N) Yang-Mills theory reduced to 0+1 dimensions

  - X = NxN matrices: reflect non-commuting short-distance structure of space-time
  - Non-local; space-time is approximate, derived concept
  - Infinite momentum frame: not manifestly Lorentz covariant
  - Large-N Limit: gives D=11 supergravity, graviton scattering

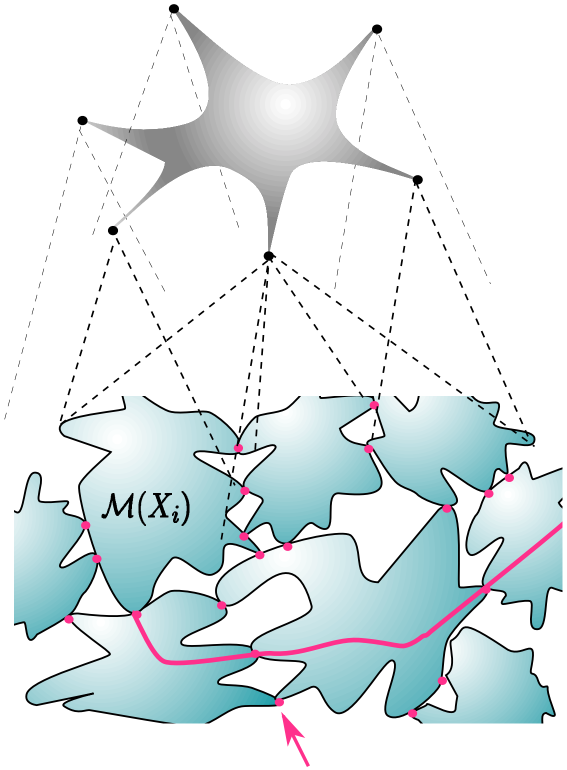
- Compactifications (eg on tori) reproduce known facts about the five D=10 string theories and their compactifications... highly non-trivial !

  Seem to provide non-perturbative formulation of type IIA and other string models.

- Incompletely understood, involves new concepts beyond quantum field theory and General Relativity .....

The Grand Picture II: N=2 SUSY Strings in D=4

All vacua are **connected** by non-perturbative transitions, and so form a complicated web with (10^8?) components (with in general different dimensions, say 100) :



D=10/11 Theories

Compactifications

Connected web
of D=4 Theories

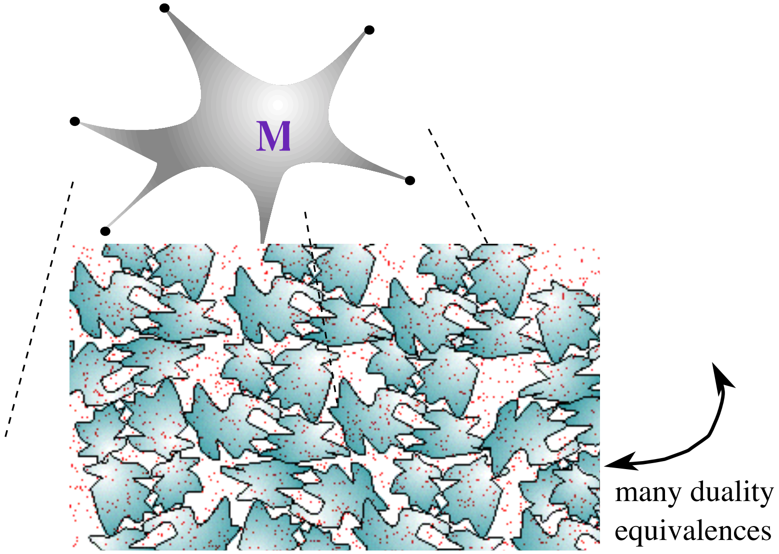
Each blob corresponds to
a family of perturbative
vacua

Connected via singular transitions:
massless states open up new branches

Different kinds of singularities give rise to many kinds of known, as well as novel physical phenomena in D=4....

N=1 Supersymmetric Theories ?

- We are beginning to investigate N=0,1 SUSY strings in D=4, which it is a problem of enormous complexity ....



- Can we still hope for a single unique vacuum state?
  Nobody knows....

> Despite all complexity: it seems that what crystallizes here is just **one single** theory, with many many facets.....

It may be that this is just the space of all possible consistent quantum theories that include gravity.........

Tests, Applications & Recent Developments

W. Lerche, ISTAPP 2011
Istanbul Part 4

- **Duality** is an extremely useful tool for analyzing in detail many non-trivial string and field theories.
  Supersymmetry facilitates this by virtue of its non-ren. properties, but is perhaps by itself not a fundamentally important feature.

- How do we know that these ideas are correct and make any sense at all ?

  Even though string theory makes **infinitely many predictions**, it is hard to verify with present day experiments ....

Theoretical Experiments: Consistency Checks

- Besides growing circumstantial evidence with varying degree of rigor, there have been numerous non-trivial quantitative tests and consistency checks.

> Not a single test of the dualities has ever failed !

- Apart from aiming for grand unifications, there has also been a lot of highly non-trivial results for gauge and other QFT's !

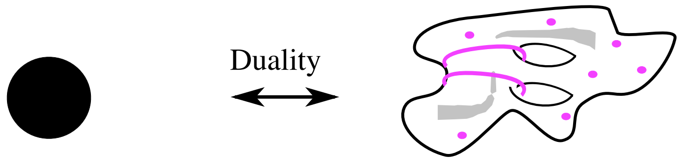
Field theoretical applications

Counting Black Hole Microstates with D-branes

- Example:
  Compute Bekenstein-Hawking entropy (= area of BH horizon)
  of extremal N=4 supersymmetric black hole in D=5.

$$S_{BE} \equiv \log(d) = 2\pi\sqrt{\frac{q_h {q_f}^2}{2}}$$ q_f, q_h = electric and axionic charges

- Idea:



Large, semi-classical black hole

Type IIB string on K3xS^1

Strongly coupled string theory ⟷ Weakly coupled string theory

For large circle, this maps to a 2d sigma model
on the moduli space of a gas of D0 branes on K3
Counting states in this sigma model,
indeed reproduces the above Bekenstein-Hawking
entropy for large charges !

This does not only add credibility to the duality claims, but also tells that there are no "missing" degrees of freedom !

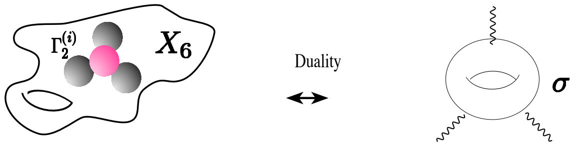
String theory seems to have exactly the right degrees of freedom to make sense of quantum black holes.....

Comparison of Quantum Corrections

- Complicated **perturbative corrections** to effective actions can computed in various different string models, and always give the same answer.

- Example: threshold corrections to gauge coupling in N=2 SUSY in D=4

Type IIA on some Calabi-Yau X$_6$ versus heterotic string on K$_3$xT$_2$



Counting spheres in the
Calabi-Yau via mirror symmetry
(tree level)

perturbative one-loop
diagram
("Borcherds integral")

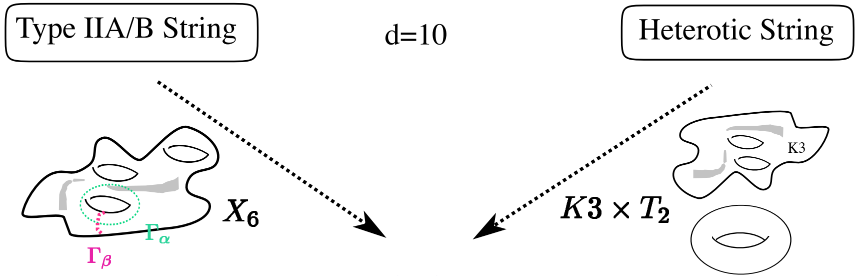
$$\tau(T,U) = \sum c_{n,m} \log[1 - e^{-nT} e^{-mU}] \qquad \tau(T,U) = \int \frac{d^2\sigma}{\mathrm{Im}\sigma^2} \mathcal{B}(T,U,\sigma)$$

These very non-trivial functions, of completely different origin, match completely !

$$\partial_T \tau_{TT}(T,U) = \frac{i}{2\pi} \frac{E_4(T)E_4(U)E_6(U)(E_4(T)^3 - E_6(T)^2)}{E_4(U)^3 E_6(T)^2 - E_4(T)^3 E_6(U)^2}$$
$$\partial_U \tau_{TT}(T,U) = -\frac{i}{2\pi} \frac{E_4(T)^2 E_6(T)(E_4(U)^3 - E_6(U)^2)}{E_4(U)^3 E_6(T)^2 - E_4(T)^3 E_6(U)^2} + \frac{i}{2\pi} \partial_T \ln f_y(q_1, q_3)$$

Recovering Seiberg-Witten Theory from String Duality

Non-perturbative equivalence of type IIB string, compactified on Calabi-Yau manifold X_6, with heterotic string compactified on K_3xT_2



Type IIA/B String

d=10

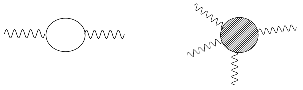
Heterotic String

Identical N=2 SUSY theory in d=4

3-branes wrapping 3-cycles Γ_α	⟷	fundamental gauge bosons
3-branes wrapping dual 3-cycles Γ_β	⟷	SW monopoles
Tree-level physics in gauge sector	⟷	Non-perturbative quantum theory
world-sheet instanton effects	⟷	space-time gauge instanton effects

classical computation ➡ exact non-perturbative SW theory, incl gravity corrections !

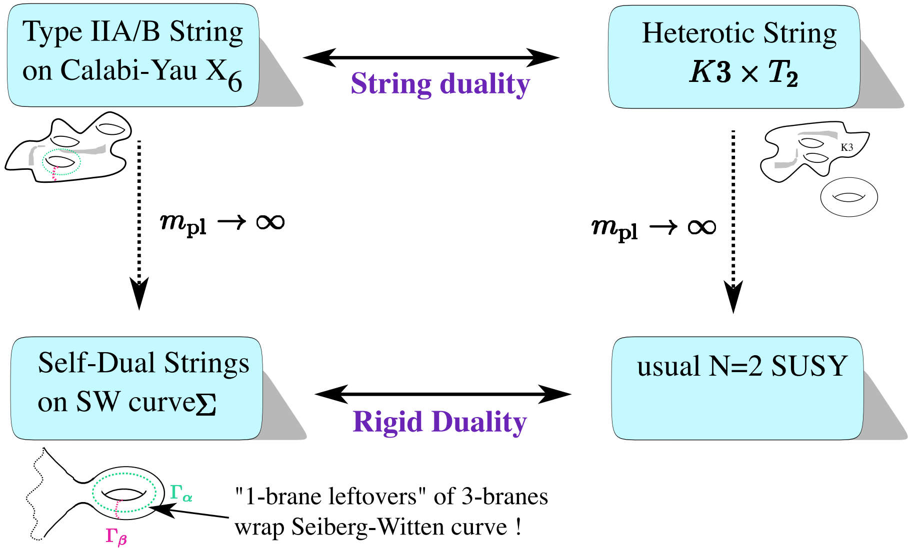
$$\tau_{\rm eff}(t) = \partial_\tau \int_{\Gamma_3} \Omega^{(\rm CY')}(t) \qquad \tau_{\rm eff}(\phi) = \frac{1}{\pi}\log\left[\frac{\phi^2}{\Lambda^2}\right] - \frac{1}{\pi}\sum_{\ell=1}^{\infty} c_\ell \left(\frac{\Lambda}{\phi}\right)^{4\ell}$$



Stringy Interpretation of Seiberg-Witten Curve Σ

We thus have a natural dual reformulation of N=2 supersymmetric Yang-Mills theory !

It is nothing but a "rigid remnant" of the type II/heterotic string duality, that remains after **decoupling gravity**:



$$\begin{tikzcd} \text{Type IIA/B String on Calabi-Yau } X_6 \arrow[r, leftrightarrow, "\textbf{String duality}"'] \arrow[d, dotted, "m_{\rm pl} \to \infty"] & \text{Heterotic String } K3 \times T_2 \arrow[d, dotted, "m_{\rm pl} \to \infty"'] \\ \text{Self-Dual Strings on SW curve}\Sigma \arrow[r, leftrightarrow, "\textbf{Rigid Duality}"'] & \text{usual N=2 SUSY} \end{tikzcd}$$

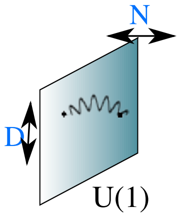
"1-brane leftovers" of 3-branes wrap Seiberg-Witten curve !

Gauge bosons and monopoles are on equal footing !
They simply correspond to **self-dual strings** wrapping Σ
Γ_β Γ_α

Elementary gauge bosons, but solitonic monopoles (or vice versa)

Can study non-perturbative properties of the N=2 gauge theory, that are extremely hard to get at in ordinary local QFT !

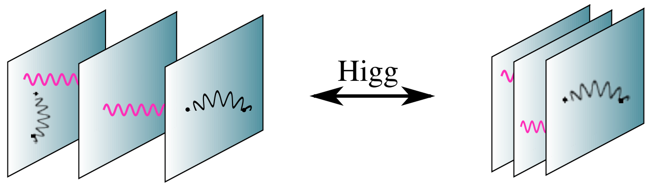
Further Applications: D-Brane Technology



- Recall single D-brane: open strings can end

Localized degrees of freedom:

D: U(1) gauge degrees of freedom A_μ
N: Higgs field, VEV $\langle \Phi \rangle$ = brane position

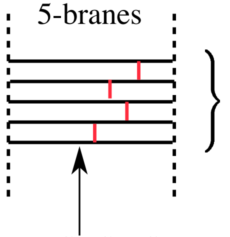


U(1) x U(1) x U(1) + massive U(3) unbroken gauge symmetry

- Decouple gravity and irrelevant string modes: get results from string duality also for **ordinary QFT**

"**D-brane technology**" can be used to model local string geometries which realize, for example, gauge theories with matter.

- Example: In Type IIA string theory, consider



n D4-branes,
open strings between them
give gauge fields

This induces N=2 SUSY SU(n) SYM theory on the world-volume of the D4-branes, reproducing Seiberg-

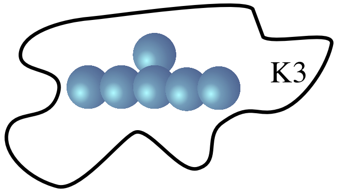
---

Geometric Engineering

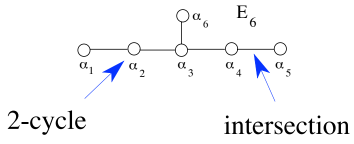
- Instead of **flat** D-branes, we can also use **curved** D-branes wrapped around p-cycles in some Calabi-Yau manifold.

The **intersection topology** determines then gauge group and matter content; it is a very systematic construction which allows to design a huge class of gauge theories, as well as novel QFT.

- Example: In Type IIA string theory on K3, consider D2-branes wrapped around 2-cycles that intersect in a

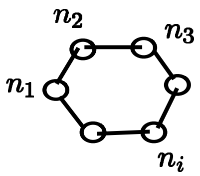


Group theoretical Dynkin diagram:



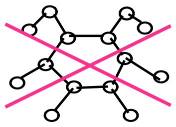
Physically, this yields an E6 gauge theory in D=6 !

- Example: "**Quiver**" N=1 SUSY gauge theory in D=4 each node corresponds to some wrapped D branes:



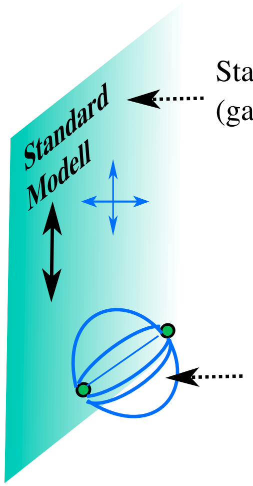
Physically, this yields a gauge group $G = \prod_i U(n_i)$ with matter fields in the reps. $(n_i, \bar{n}_{i+1})$

- Not everything is allowed:



Low-Scale Strings ?

- This setup may be phenomenologically very interesting:



Standard-model localizes on the brane volume (gauge interactions and matter fields: open strings)

Gravity propagates also in the "bulk" off the brane (closed strings)

- The gravity field lines spread out to more than 4 dimensions, and are "diluted" : gravity appears in the brane world **weaker** than in the bulk !

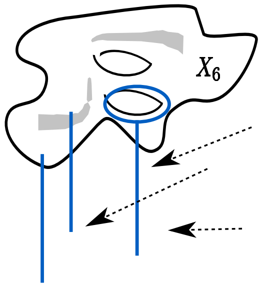
$$G_N \sim \frac{1}{M^2}$$

This means the scale of gravity (hitherto 10^{19}GeV) can be much smaller, in fact as low as the scale of the weak interactions, or even smaller..

If true, this could be tested at the LHC !

Stringy "Brane Worlds"

- A very general class of N=1 supersymmetric backgrounds can be obtained by placing extra D-branes on a compactification space, eg on a Calabi-Yau space in type II string theory:



D-branes partically wrapping cycles or sitting at points on X6

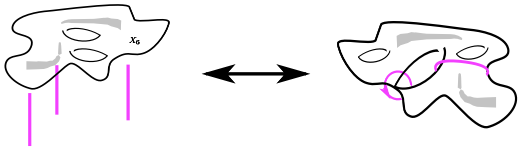
N=1 SUSY theory in 3+1 dimensions

This represents a "**brane world**" where we live on the 3+1 dimensional "left-over" of the D-branes

Generically, this picture is dual to strongly-coupled heterotic strings on large compact dimensions (as naturally suggested by SUSY breaking)

- Note however, that due to stringy geometry, a geometrical interpretation is in general highly ambiguous ...

One and the same effective action may have many different dual interpretations:

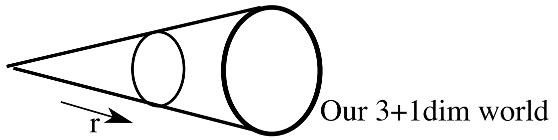


Model Building with "Warped" Geometries

- A related approach rests on the fact that branes can induce a "warped" space time, which is not a direct product of R^4 and an internal space; rather a fiber product with metric:

$$ds^2 = e^{-f(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2$$

Visualize as a cone:

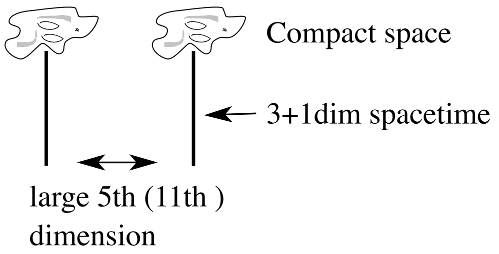


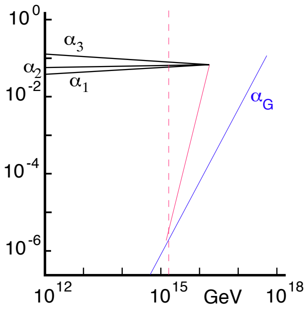
moving along the 5th dimension changes energy in 3+1dim

A warp factor corresponding to a large 5th dimension can make gravity appear much weaker in our 3+1 dimensional world than it is in the "bulk" 5th dimension.

- Horava-Witten scenario

Strong coupling limit of 10dim heterotic string gives 11dim M-theory "compactified" on a line interval, bounded by two "end-of-the-world" branes.

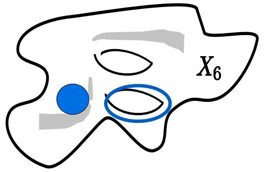




Flux Vacua

- Apart from branes that can wrap around p-cycles of a CY, there can be also “trapped” flux of the tensor gauge fields:

$$N_p^i = \int_{\gamma_p^i} F_p, \quad F = dC$$



This contributes to the superpotential in the form

$$W(t) = \sum_{i,p} N_p^i \, \Pi_i(t)$$

periods

Since the number i(p) of p-cycles is typically large (100’s), there appears a large numer of of possible combinations of the discrete flux numbers; a typical estimate of consistent vacua is 10^{500}.

So on top of the choice of CY/branes/moduli, there is an enormous number of possible flux configurations one can have, dramatically worsening the landscape problem!

However, sometimes a curse can turn out to be a blessing....

Anthropic Cosmological Constant

- One of the most serious problems is to understand the observed value of the cosmological constant (vacuum energy) which is exceedingly small:

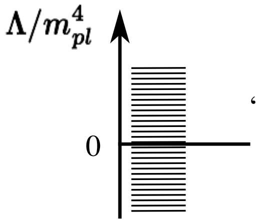
$$\Lambda = 10^{-120} m_{pl}^4$$

This is by far the worst of all hierarchy problems!

What physical principle can possibly generate such a small scale?

The enormous number of flux vacua allows for a natural, heuristic explanation...

Due to their combinatorics, one can show that invariably there will always exist a number of flux vacua which lead to the correct value of the c.c:



“Discretuum”

> This is not really a prediction but anthropic reasoning, and thus has spurred a lot of debate whether this would be a scientifically valid strategy. However, until today this is the only known approach that would be able to “explain” the value of the c.c.

KLLT Szenario

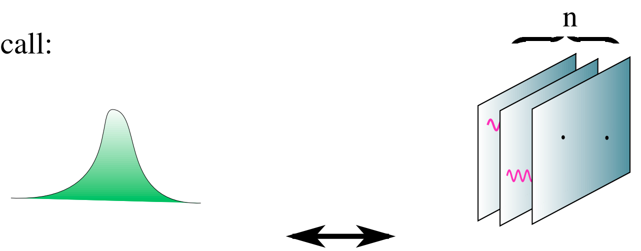
- moduli stabilization by fluxes and non-pert effects
- SUSY breaking
- de Sitter vaccum with realistic c.c.
- self-consistent, semi-controllable setup

moduli stabilization by fluxes and non-pert effects

features

Duality between Open and Closed Strings

- Recall:



Closed string (Type II)
p-brane solitons

non-perturbative;
involves gravity and
flux number n

Open string (Type I)
elementary D-p-branes

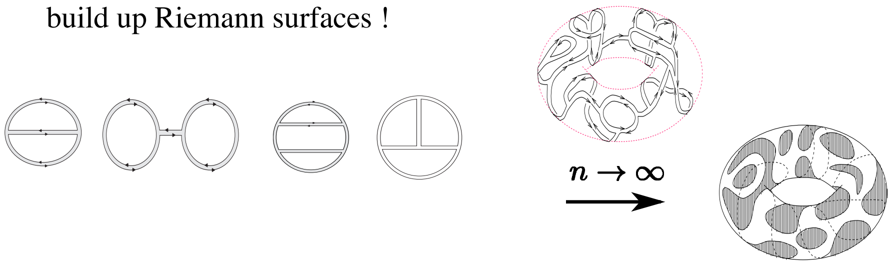
perturbative;
U(n) gauge theory

Is there a deeper relationship between the closed string (gravity) sector, and the open string (gauge theory) sector ?

- An old idea by t'Hooft:

Consider a U(n) gauge theory, and take the limit $n \to \infty$

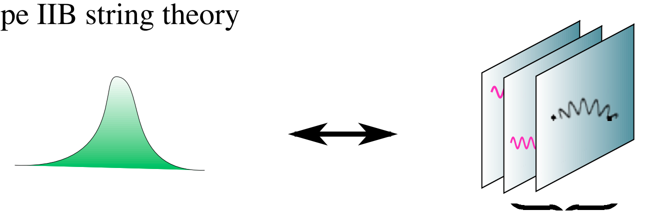
Then only "planar" Feynman digrams dominate, and in fact build up Riemann surfaces !



- The conjecture was that for large n, the gauge theory turns into some sort of string theory - but exactly which one wasn't clear at all, for a long time.

The AdS Correspondence: An exact duality between Gauge and String Theory

- Type IIB string theory



n D3-branes
yield U(n) N=4 SUSY on the
3+1 dimensional brane volumes

Geometry effectively

$S^5 \times AdS_5$

$\sum_{i=1}^{6} x_i{}^2 = R^2$ $\qquad$ $x_0{}^2 + x_1{}^2 - \sum_{i=1}^{5} x_i{}^2 = R^2$

"anti de Sitter space"

- Claim (Maldacena):

The large-n limit of the U(n) N=4 SUSY gauge theory is exactly dual to the type IIB string on $S^5 \times AdS_5$

Correlators of gauge invariant operators in the N=4 gauge theory are 1:1 to the Green's functions of the type IIB string... many tests !

This corresponds to a weak-strong coupling S-duality:

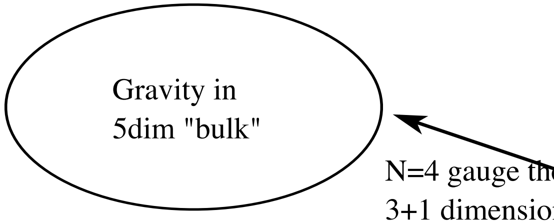
String theory weakly coupled for:		Gauge theory weakly coupled for:	
$g^2 n$	large	$g^2 n$	small
g	small	g	small

- Using this correspondence, many interesting results about Yang-Mills theory were obtained, including theories with less or no SUSY....

$$V(r) \sim g_{YM}\sqrt{n}/r$$

The Holographic Principle

- Geometry of the AdS-space:



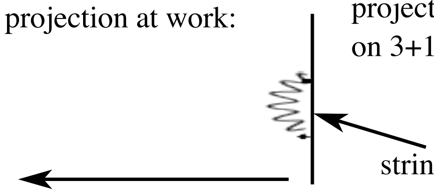
Gravity in
5dim "bulk"

N=4 gauge theory on the
3+1 dimensional boundary
=Minkowski space

How can a theory be equivalent to "another" theory on the boundary ?
It seems to have too many degrees of freedom !

> The "Holographic Principle:"
>
> A theory of quantum gravity is non-local and carries the same number of degrees of freedom as a theory on the boundary !
> (inspired by black holes)

- Holographic projection at work:



projection
on 3+1 dim

string in the bulk maps
to QCD string between
quarks on the boundary

5th dimension
(energy)

(similar features seen in warped string compactification models)

A Holographic Universe ?

- The gauge/gravity correspondence for N=4 gauge theory has been the first example in which the holographic principle is realized, and this has been checked thoroughly.

Thus, another example where an investigation of string or brane physics had a unexpected, stunning outcome....

- Could it be that these ideas apply also to our universe as a whole ?

  There are some indications for this, but no clearcut conclusions so far - some, very speculative, ideas are very interesting; for example, that the evolution of our universe between big bang and big crunch may be something like a renormalization group flow between two conformal fixed points.......