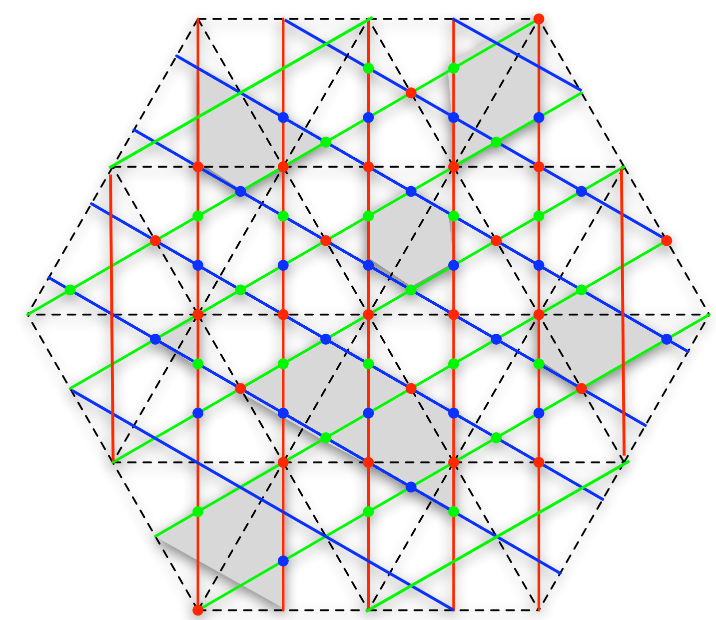
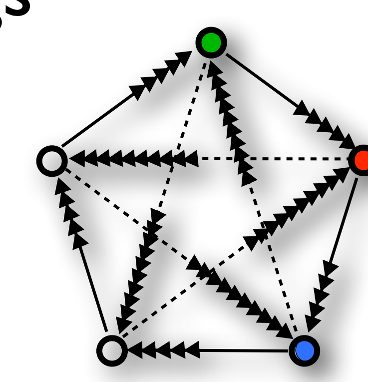
Matrix Factorizations and Homological Mirror Symmetry

W.Lerche, TSIMF 1/2019

arXiv:1803.10333

- Motivation: quantum geometry of general D-brane configurations
- Recap: closed string mirror symmetry
- LG models: contact terms vs. flat coordinates
- Open string = homological mirror symmetry
- Matrix factorizations and their deformations
- Open string mirror map from super-residue pairings
- Example



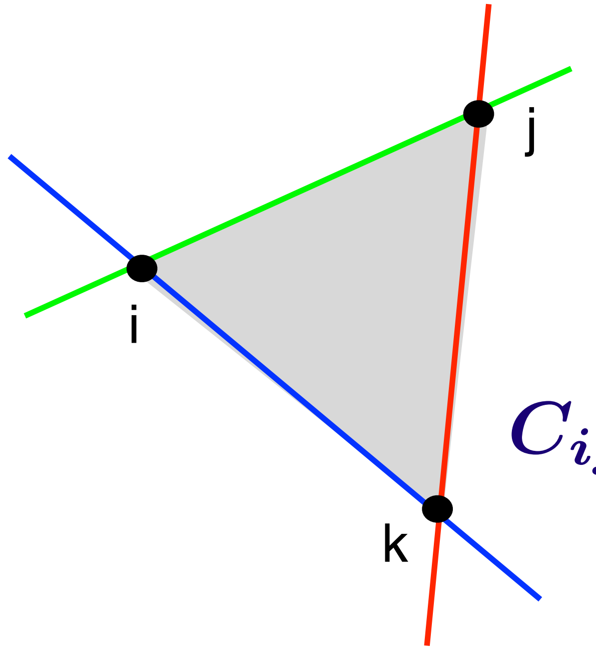


Physics of intersecting brane geometries

- Open string mirror symmetry is by far not as well developed as for closed strings!

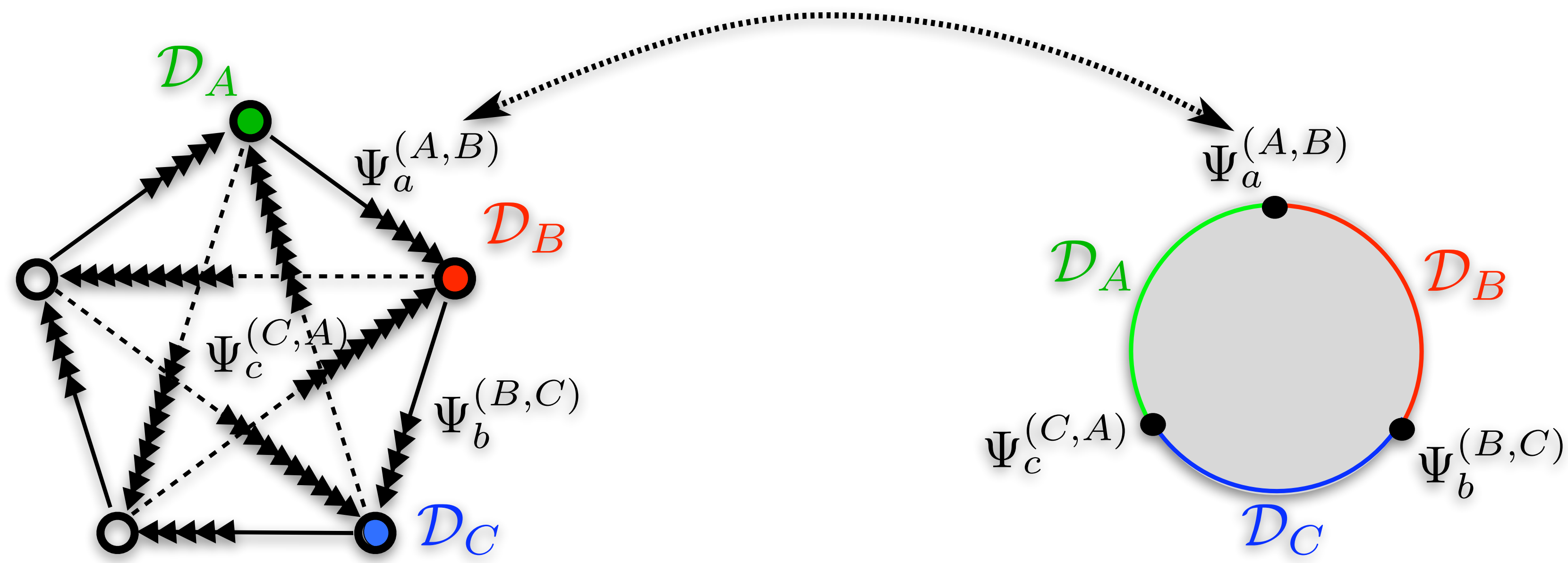
So far, mostly non-generic (non-compact, non-intersecting) brane configurations were considered; almost nothing has ever been computed for intersecting branes eg. on Calabi-Yau threefolds.

- Phenomenological interest:
  - Chiral fermions
  - Exponentially suppressed Yukawa's



$$C_{ijk} \sim e^{-\mathbf{Area}}$$

Effective superpotential for quivers



Quiver diagram

Disk world sheet in TCFT

F-term superpotential ~ closed paths in quiver

$$\mathcal{W}_{eff}(T,u,t) = T_a T_b T_c \underbrace{\langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,A)} \rangle}_{C_{abc}(t,u)} + T_a T_b T_c T_d \underbrace{\langle \Psi_a^{(A,B)} \Psi_b^{(B,C)} \Psi_c^{(C,D)} \Psi_d^{(D,A)} \rangle}_{C_{abcd}(t,u)} + \ldots$$

space-time fields

closed and open string moduli $\sim \mathrm{const} + \mathcal{O}(e^{-t}, e^{-u})$

instanton corrections = open GW invariants: how to compute?

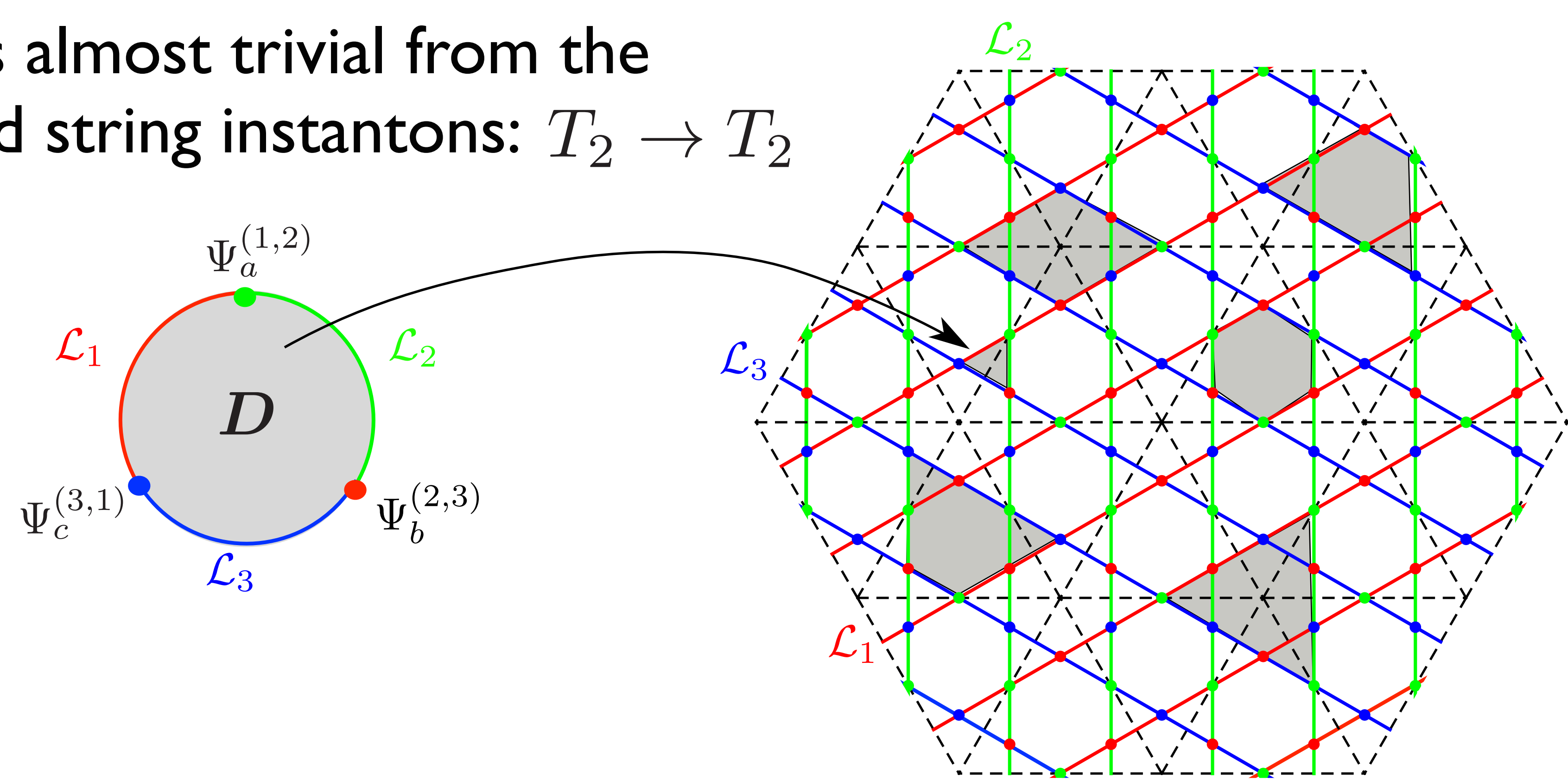
Math: Homological mirror symmetry

Kontsevich 1994

- Open string mirror symmetry becomes (really) non-trivial for intersecting branes

  There is an infinitely richer diversity of world-sheet instantons, ie., Gromov-Witten invariants.

- Eg. the elliptic curve is almost trivial from the point of view of closed string instantons: $T_2 \to T_2$



However in the open string sector with intersecting branes, an arbitrary number of polygon-shaped disk instantons may contribute to the superpotential!

Lightning recap: closed string mirror symmetry

Type IIA String on Calabi Yau Y ⟺ Type IIB String on Calabi Yau X

- Moduli space of N=2 vector SM: $\mathcal{QM}_K^{h_{1,1}}(Y,t) \simeq \mathcal{M}_{CS}^{h_{2,1}}(X,z)$
- 3-pt functions:

$$C_{klm} = \int_Y J_k \wedge J_l \wedge J_m + \sum_{d_1,\ldots,d_k} \frac{n^r_{d_1,\ldots,d_k} d_k\, d_l\, d_m}{1-\prod_{i=1}^k q_i^{d_i}} \prod_{i=1}^k q_i^{d_i} \longleftrightarrow \frac{p_{abc}(z)}{\prod \Delta(z)} \frac{\partial z_a}{\partial t_k} \frac{\partial z_b}{\partial t_l} \frac{\partial z_c}{\partial t_m}$$

A-model: deformed quantum geometry from world-sheet instantons = holom maps $P_1 \to Y$, $q = e^{-t}$

B-model: classical geometry

- Mirror map:

$$t_i := \int J_i^{1,1}(Y) + \ldots \longleftrightarrow \int_{\gamma_a^3} \Omega^{3,0}(X) =: \ln z_a(t) + \mathcal{O}(z)$$

flat coordinates on $\mathcal{QM}_K^{h_{1,1}}(Y)$ ⟺ Period integral = flat coo on $\mathcal{M}_{CS}^{h_{2,1}}(X)$

Math: Gauss-Manin system

- The period integrals satisfy certain flatness diff. equations that arise from the variation of Hodge structures.

  Essentially this boils down to a linear system of the form

$$\nabla \cdot \Pi \equiv \left(\delta_j^k \partial_{t_i} + (C_i)_j{}^k - (\Gamma_i)_j{}^k \right) \begin{pmatrix} \int \frac{1}{W} \\ \vdots \\ \int \frac{\phi^\lambda}{W^{\lambda+1}} \end{pmatrix}_k = 0$$

Yukawa's/ring OPE coeffs

Gauss-Manin connection

period vector ∏

Calabi-Yau defined by $X : W(x_i, z) = 0$

- $\Gamma = 0$ defines flat coordinates (and thus the mirror map): $z = z(t)$

  … as well as flat operator bases via $\phi_i(x,t) = \partial_{t_i} W(x, z(t))$

Physical realization: superconformal B-twisted TCFT

All this has a concrete realisation in field theoretical models:

- W(x,z) is the superpotential of a N=(2,2) Landau-Ginzburg model

$\phi_i(x,t) = \partial_{t_i} W(x,z(t))$ forms a flat basis of the chiral ring

$$\langle \phi_k \phi_l \rangle = \mathbf{const.}$$

- In terms of these, all correlators are given in terms of residue integrals:

$$C_{klm}(t) \equiv \langle \phi_k \phi_l \phi_m e^{\int t_i \phi_i^{(2)}} \rangle = \oint \frac{1}{(dW(x,t))^N} \phi_k(x,t) \phi_l(x,t) \phi_m(x,t)$$

$$= \partial_{t_k} \partial_{t_l} \partial_{t_m} \mathcal{F}(t) \quad \text{integrability}$$

$$C_{klmn_1..n_r}(t) = \partial_{t_{n1}} ... \partial_{t_{nr}} C_{klm}(t)$$

"Special Geometry"

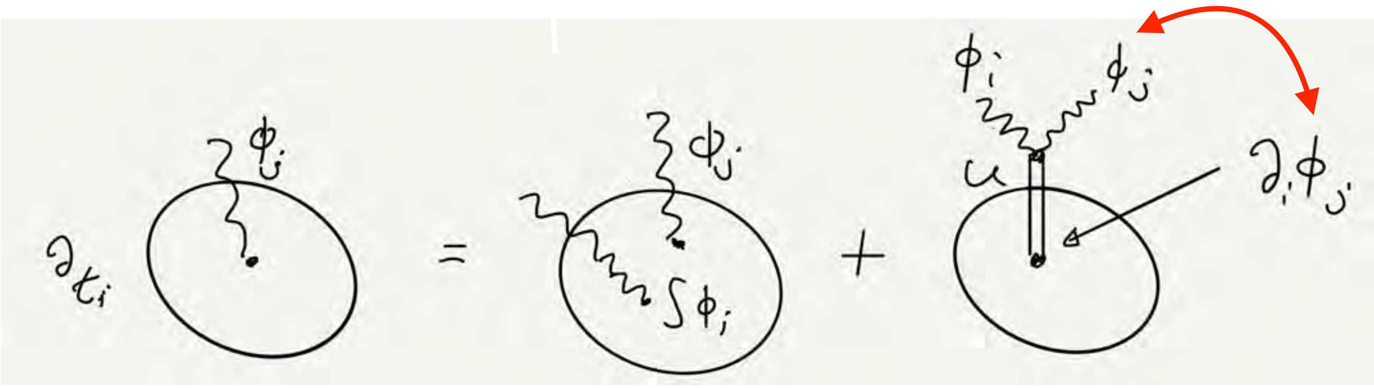
Math-Phys: Contact terms versus flat coordinates

- The Gauss Manin eqn. encodes contact terms:

$$0 = \Gamma = \partial_{t_i}\phi_j - U(\phi_i\phi_j)$$

where U plays the role of the closed string propagator

$$U(\mathcal{O}(x,z)) \equiv d_{x_k}\left(\frac{\mathcal{O}(x,z)}{d_{x_k}W(x,z)}\right)_+ \sim \frac{G_0\bar{G}_0}{L_0}\mathcal{O} \qquad \mathcal{H}_E \to \mathcal{H}$$



- Functional dependence reflects renormalization by iteratively integrating out massive fields:

$$\phi(t) = \phi(0) + t\,U(\phi\phi) + 1/2\,t^2\,U(\phi\,U(\phi\phi)) + \cdots = \partial_t W(x, z(t))$$

Summing up all nested trees in one swoop!

Saito's higher residue pairings

- Reformulate by avoiding period integrals while emphasizing contact terms:

  Localize path integral with insertion $e^{-\lambda(L_0+uU)}$ for $\lambda \to \infty$

  produces residue pairings $K[u](\phi_k,\phi_l) \equiv \sum_{\ell\geq 0} u^\ell K^{(\ell)}(\phi_k,\phi_l)$

  where u is a spectral parameter that counts the number of c.t. and

$$K^{(\ell)}(\varphi_k,\varphi_l) = \oint \frac{dx}{(dW)^N} \sum_{n=0}^{\ell} (-1)^{\ell-n} \overbrace{U(U(\ldots U(}^{n}\varphi_k)..)\overbrace{U(U(\ldots U(}^{\ell-n}\varphi_l)..)$$

- In terms of these, the Gauss-Manin diff eqs can be written compactly:

$$K^{(0)}(\varphi_k,\varphi_l) = \eta_{kl} = \mathrm{const}, \qquad K^{(\ell>0)}(\varphi_k,\varphi_l) = 0,$$

$$K[u](\nabla_t\varphi_a,\varphi_b) = K[u](\varphi_a,\nabla_t\varphi_b) = 0, \qquad \nabla_t \equiv \partial_t - \frac{\partial_t W}{u}$$

More explicitly….

- Consider elliptic curve where $\Pi = \left(\int \frac{1}{W}, \int \frac{\phi}{W^2}\right)$

  K samples all components of the Gauss-Manin connection:

$$\langle \Pi_i, \nabla \Pi_j \rangle \equiv \langle \Pi_i, (\partial + C - \Gamma)\Pi_j \rangle$$

$$= \begin{pmatrix} K(\phi, \nabla 1) & K(1,\phi) - 1 \\ K(\phi, \nabla \phi) & K(1, \nabla \phi) \end{pmatrix}_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{ij}$$

$$K^1(\phi, \partial\phi) - K^2(\phi, \phi\phi)$$
$$= K^0(\phi, U(\partial\phi - U(\phi\phi)))$$

$$K^0(1, \partial\phi) - K^1(1, \phi\phi)$$
$$= K^0(1, \partial\phi - U(\phi\phi))$$

These inner products can be easier generalized to open strings where the fields become matrices

Now on to open strings…

- Mirror symmetry between A- and B-models

  ⟹ Homological mirror symmetry between categories of A- and B-type branes

- Hodge theory of CY-spaces

  ⟹ Non-comm. Hodge theory on A_∞ categories

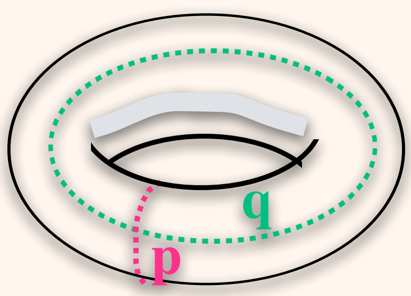
- LG field theoretical realisation based on W

  ⟹ Boundary LG model based on Matrix factorizations of W

Homological Mirror Symmetry for Poor Physicist

- Mirror symmetry acts between full categories descr. A- and B-branes!

A-Model on Y



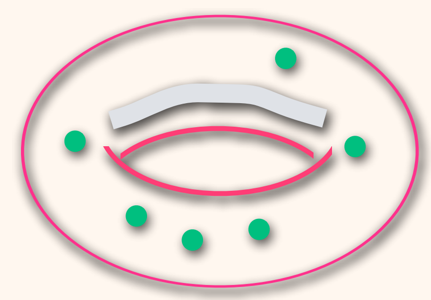
D1 branes on (p,q) cycles

..."Fukaya category" of lagrangian cycles on Y

mirror symmetry

⟷

B-Model on X



$(N_2,N_0) = (r,c_1)$ of gauge bundle

..."bounded derived category" of coherent sheaves on X

$$Fuk(Y) \quad \longleftrightarrow \quad D^b(Coh(X))$$

- There is much more to this than just quantum numbers (K-theory), or isomorphisms between categories

Open string mirror symmetry

- Irrespective of fancy maths, the problem can be formulated entirely in terms of physics (and we define HMS this way)

  Math isomorphisms ⟹ Equality of infinitely many correlators, = A_∞ products

- Consider deformations by closed string perturbation t

  Closed string mirror map:

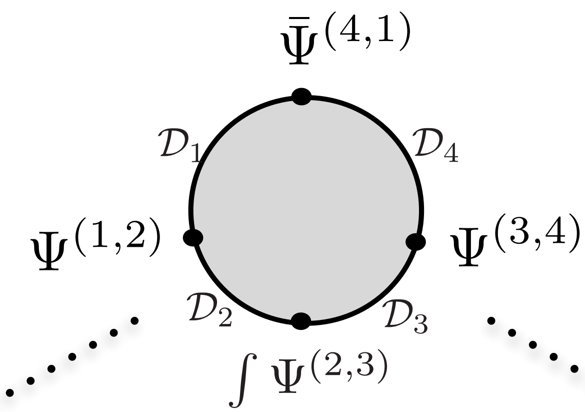
  $$\begin{array}{ccc} \text{A-model} & & \text{B-model} \\ t(z) & \longleftrightarrow & z(t) \end{array}$$

  Open string: $$Fuk(Y) \longleftrightarrow D^b(Coh(X))$$

- How to tie together explicitly?
  .. impose appropriate flatness eqs playing the role of mirror map

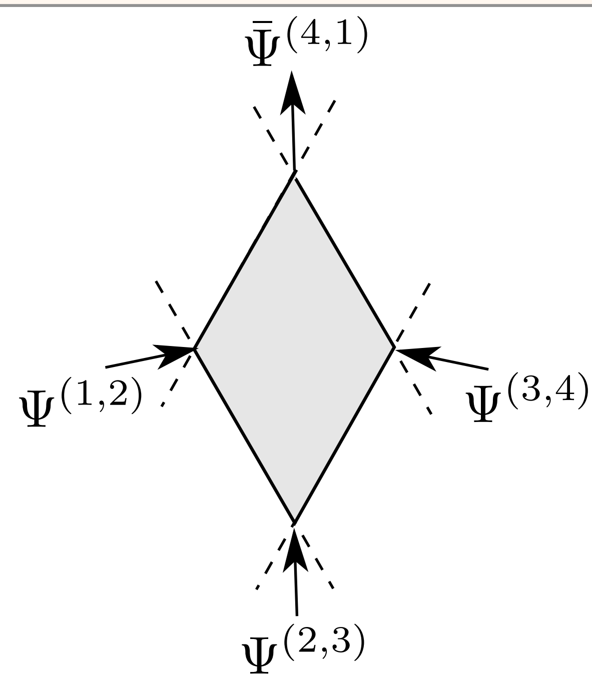
Mirror symmetry of A∞ products

$m_3(\Psi^{\otimes 3})$

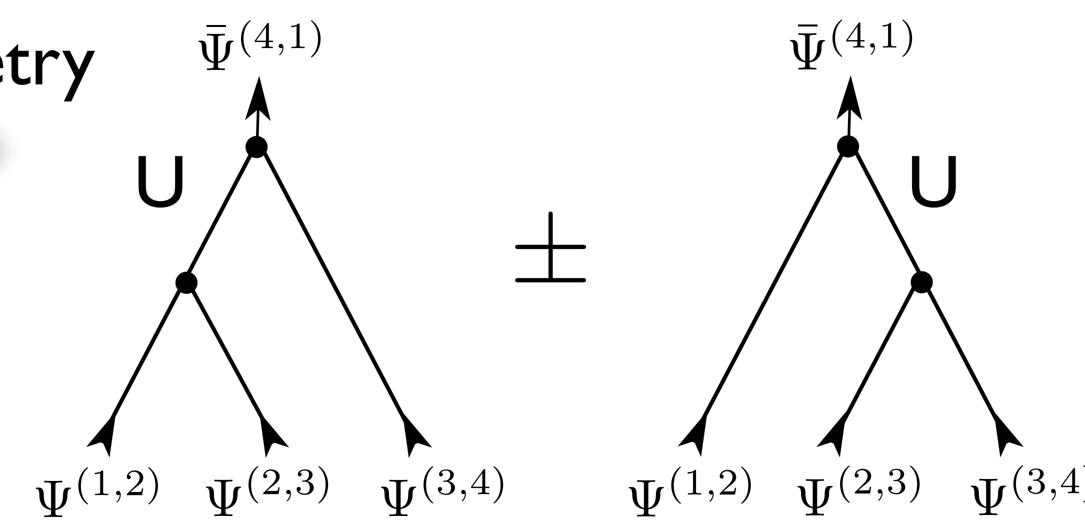


A-Model
localizes on holomorphic maps:
world-sheet instantons $D \to Y$

B-Model
localizes on constant maps:
nested trees



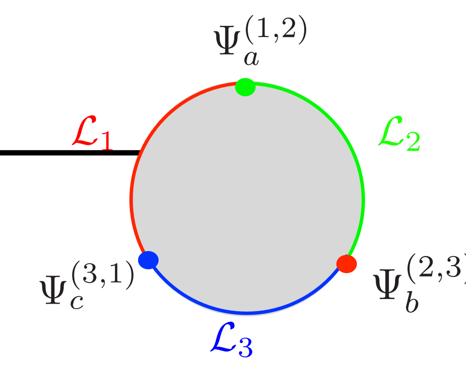
mirror symmetry



quantum Fukaya product $m_3 \sim e^{-S_{inst}}$

classical Massey product

String correlators and A∞ products

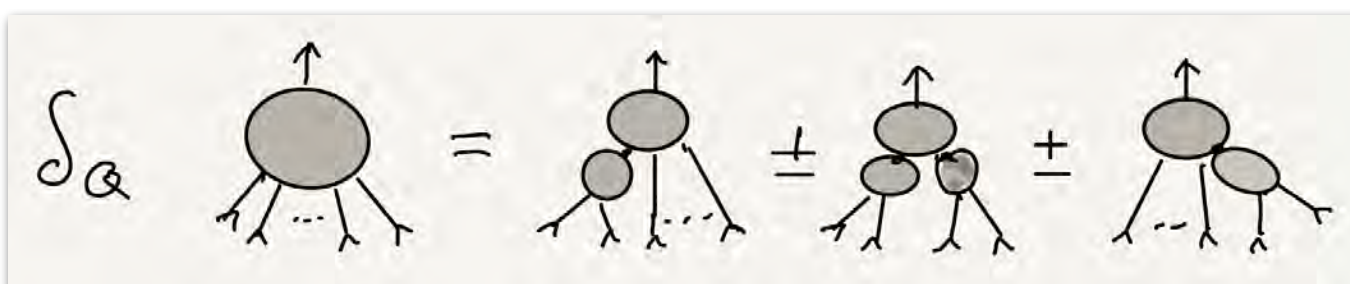


$$C_{a_0,a_1,\ldots,a_k} = \langle \Psi_{a_0} \Psi_{a_1} P \int \Psi^{(1)}_{a_2} \ldots \int \Psi^{(1)}_{a_{k-1}} \Psi_{a_k} \rangle$$
$$= \langle\langle \Psi_{a_0}, m_k(\Psi_{a_1} \oplus \ldots \oplus \Psi_{a_k} \rangle\rangle$$

- Multilinear, non-comm. maps $m_k : \Psi^{\otimes k} \to \Psi$

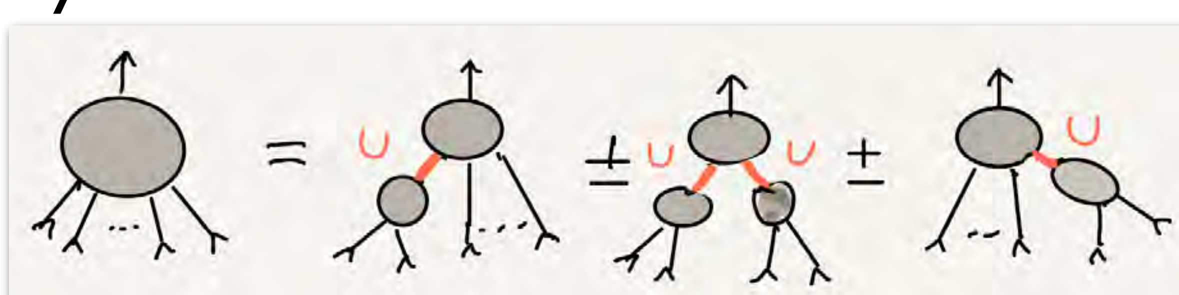
$$m_0 = 0, \quad m_1 = Q, \quad m_2 = \bullet$$

satisfy A∞ relations = Ward identities from disk factorization:



$$m_1 \cdot m_4(1,2,3,4) = m_3(m_2(1,2),3,4) \pm m_2(m_2(1,2),m_2(3,4)) \pm m_3(1,2,m_2(3,4))$$

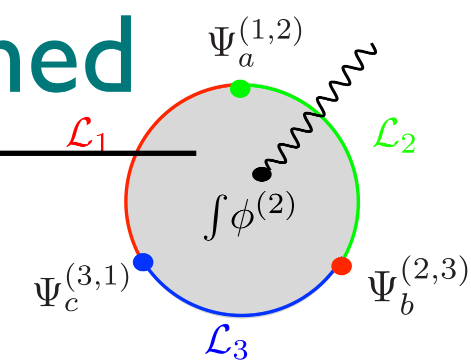
- Can be recursively solved in closed form:



U = Q[-1]
open string propagator

$$m_4(1,2,3,4) = m_3(U \cdot m_2(1,2),3,4) \pm m_2(U \cdot m_2(1,2), U \cdot m_2(3,4)) \pm m_3(1,2,U \cdot m_2(3,4))$$

String correlators and A∞ products - deformed

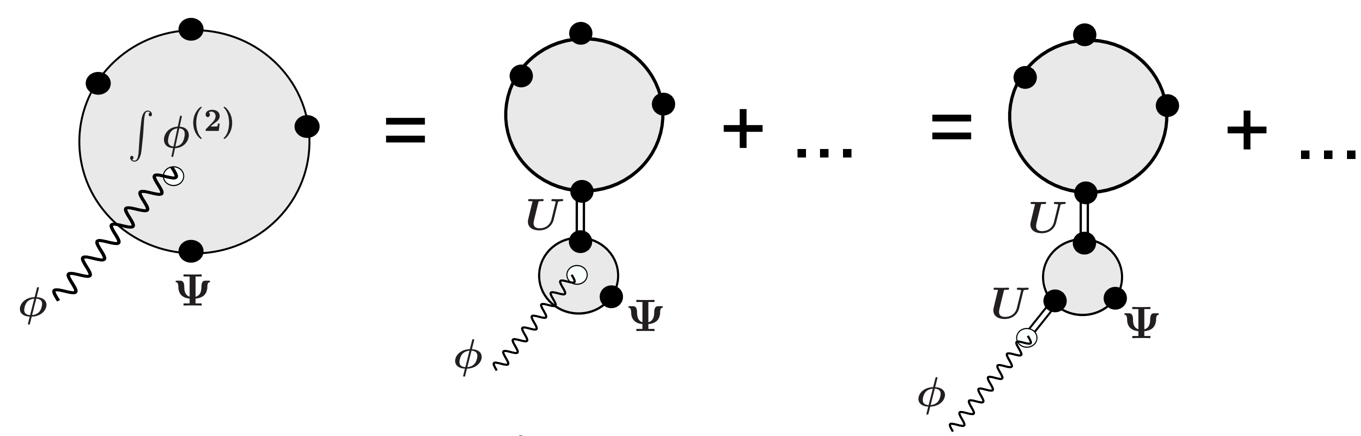


- We are interested in the dependence on bulk deformations t

$$C_{a_0,a_1,\dots,a_k}(t) = \langle \Psi_{a_0} \Psi_{a_1} P \int \Psi^{(1)}_{a_2} \dots \int \Psi^{(1)}_{a_{k-1}} \Psi_{a_k} e^{-t_k \int \phi^{(2)}_k} \rangle$$

$$= \langle\langle \Psi_{a_0}, m^t_k(\Psi_{a_1} \oplus \dots \oplus \Psi_{a_k} \rangle\rangle$$

- Deformed multilinear products satisfy "weak" A∞ relations where $m_0 \neq 0$
- Form extended structure: "open/closed homotopy algebra"



- How to compute t-dependence ?

B-type, boundary LG models: matrix factorizations

Kapustin,Li
BHLS

- Consider 2d LG model with superpotential:

$$\int_{\Sigma} d^2 z d\theta^+ d\theta^- W_{LG}(x,t) + cc.$$

(W(x,t)=0 describes CY 3-fold X)

- If there is a boundary, B-type SUSY variations induce a "Warner"-term.

  This can be cancelled by boundary dof. whose BRST operator satisfies:

$$Q(x,t,u)_{2n\times 2n}\cdot Q(x,t,u)_{2n\times 2n} \;=\; W_{LG}(x,t)\,1_{2n\times 2n}$$

- The matrices live in the Chan-Paton space and can have arbitrarily high dimension, 2n.

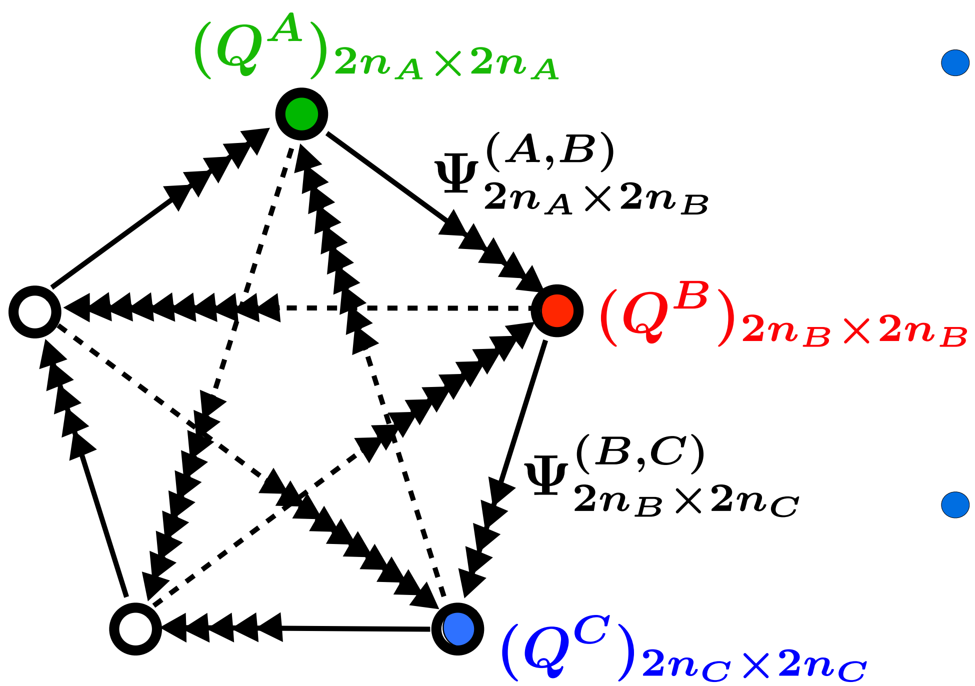
  The precise form encodes the brane geometry and depends on K-charges and possible deformation moduli t,u.

- The set of all matrix factorizations of W describes all possible B-type boundary conditions!

The category of matrix factorizations Cat(MF(W))

Math. Theorem: Kontsevich, Orlov

Cat(MF(W,X)) ~ D^b(Coh(X)), Category of coherent sheaves on X



- objects = chain complexes

$$\mathcal{P} = \left(P_1 \underset{p_0}{\overset{p_1}{\rightleftarrows}} P_0 \right) \qquad Q = \begin{pmatrix} 0 & p_0 \\ p_1 & 0 \end{pmatrix}$$

$p_{1,2}$ ~"tachyons" $p_0 p_1 = p_1 p_0 = W1$

- morphisms = boundary changing operators

$$(\Psi_a^{(A,B)})_{2n_A\times 2n_A} \in \mathrm{Ext}^1(X; \mathcal{D}^A, \mathcal{D}^B)$$

- non-triv. cohomology $\Psi^{(A,B)}: \ d\cdot\Psi^{(A,B)} = 0, \quad \Psi_a^{(A,B)} \neq d\cdot *$

where $d\cdot\Psi^{(A,B)} \equiv Q_A\Psi^{(A,B)} \pm \Psi^{(A,B)}Q_B$

- (non-comm.) composition maps $\Psi_a^{(A,B)}\cdot\Psi_b^{(B,C)} = C_{ab}{}^c\,\Phi_c^{(A,C)}$

Correlators from matrix factorizations

- Easy part:
  Construct representatives $\Psi \in \ker d/\mathrm{Im}\, d$ and recursively compute m_k:

$$C_{a_0,a_1,\ldots a_k}(t) \;=\; \langle\langle \Psi_{a_0}, m_k^t(\Psi_{a_1} \oplus \ldots \Psi_{a_k} \oplus)\rangle\rangle$$

with inner product = supertrace residue pairing

$$\langle\langle A,\, B\rangle\rangle \;=\; \oint \mathrm{str}\left(\left(\frac{d_i Q}{d_i W}\right)^{\otimes N} A \cdot B\right)$$

Kapustin-Li

- Can always choose representatives such that the two-point fct is const:

$$\langle\langle \Psi_a^{(A,B)},\, \Phi_b^{(B,A)}\rangle\rangle = \delta_{ab}$$

- Difficult part: what is the proper flat, renormalized operator basis?

$\Psi_a \to g_a(t)\Psi_a,\; \Phi_a \to g_a(t)^{-1}\, \Phi_a$ A priori freedom of rescaling….

…leaves corrs undetermined, eg: $\langle\langle \Psi, \Psi\Psi\rangle\rangle \sim g(t)^3$

Analog of Gauss-Manin eqs at the boundary?

- Generalization to non-commutative Hodge-Theory has been a major theme in math literature. Kontsevich, Pantev, Katzarkov, Sheridan, Shklyarov ,….

  However, it turned out (after much agony!) that much of these works seems almost orthogonal to what we want to do!

    - There is no degree-2 spectral parameter u at the boundary
    - Open-closed maps kill precisely the boundary changing sectors we are interested in (need more than Hochschild cohom)

- Crucial phys. extra ingredients:

    - Coupled bulk-boundary deformation problem
    - Mixed bulk-boundary contact terms
    - Generalization of residue pairings to matrix factorizations

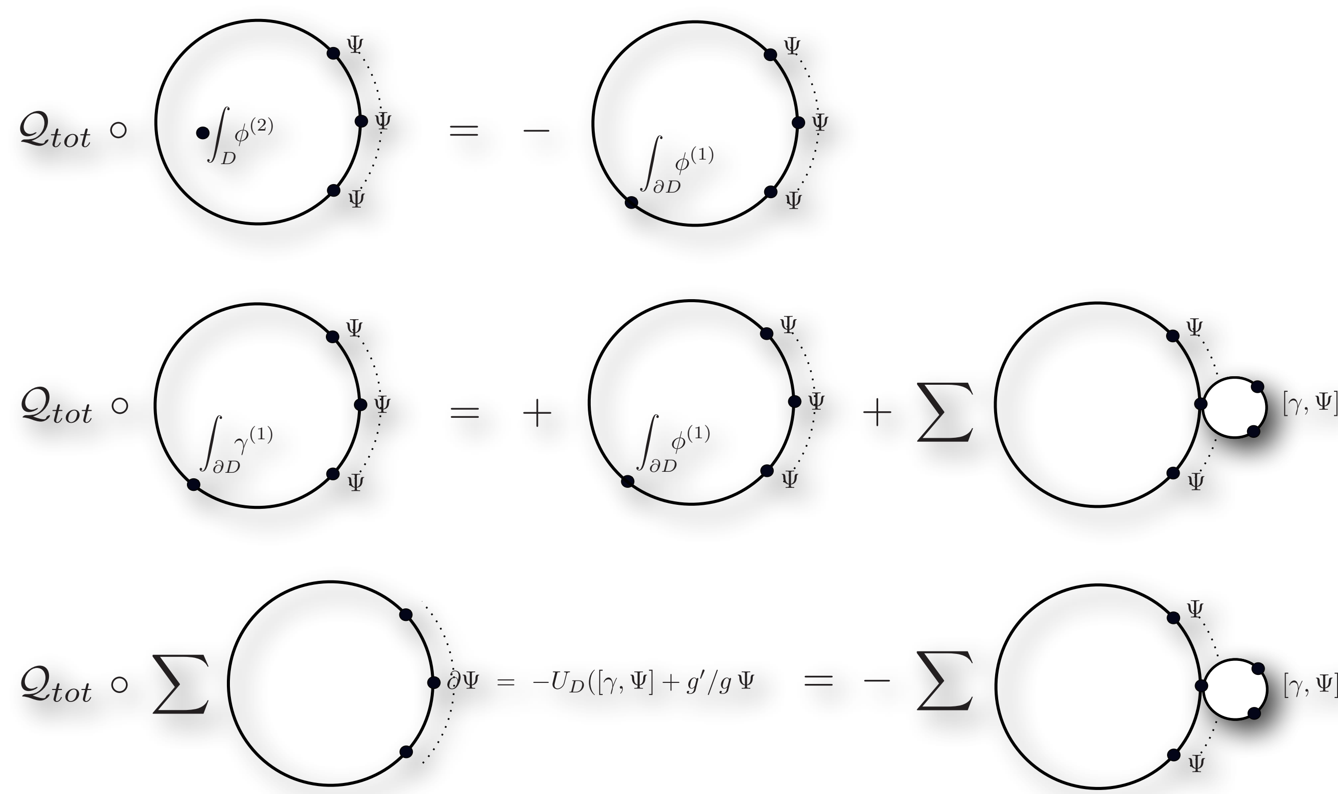
Coupled bulk-boundary deformation problem

- Due to bulk-boundary contact terms, the bulk perturbation $\phi = \partial_t W$ must be accompanied by a "Warner" boundary counter term $\gamma = \partial_t Q$

$$\delta S = t \left(\int_D \phi^{(2)} \mathbf{1} - \int_{\partial D} \gamma^{(1)} \right)$$

This combo perturbation preserves $Q(t)^2 = W(t)\mathbf{1}$ so is unobstructed. It is the natural Q-invariant pairing in relative (co-)homology of disk.

- What matters are the contacts term of γ with the other boundary ops ψ:



Higher supertrace residue pairings

- Construct higher Kapustin-Li pairings to systematically capture contact terms

$$K^{(0)}_{KL}(\Psi_a, \Phi_b) = \oint \mathrm{str}\left(\left(\frac{d_i Q}{d_i W}\right)^{\otimes N} \Psi_a \cdot \Psi_b\right)$$

$$\stackrel{!}{=} \delta_{ab} = \mathrm{const}$$

$$K^{(1)}_{KL}(\Psi_a, \Phi_b) = \frac{(-1)^{n+1}}{(n+1)!} \sum_{k=1}^{n} (-1)^{k(|\Psi_a|+1)} \sum_{i_*=1}^{n} \epsilon_{i_1 \dots i_n} \times$$

$$2 \oint \mathrm{str}\left[\left(\frac{d_{i_1} Q}{d_{i_1} W} \cdots \frac{d_{i_{k-1}} Q}{d_{i_{k-1}} W} \frac{d_k \Psi_a}{d_k W} \frac{d_{i_{k+1}} Q}{d_{i_{k+1}} W} \cdots \frac{d_{i_n} Q}{d_{i_n} W} \; \Phi_b\right)\right.$$

$$\left. - \left(\frac{d_{i_1} Q}{d_{i_1} W} \cdots \frac{d_{i_k} Q}{d_{i_k} W} \Psi_a \frac{d_{i_{k+1}} Q}{d_{i_{k+1}} W} \cdots \frac{d_{i_{n-1}} Q}{d_{i_{n-1}} W} \frac{d_n \Phi_b}{d_n W}\right)\right]$$

Instead of bosonic spectral parameter u of degree 2, we have (formally) a fermionic parameter of degree 1

$$U = \frac{G_0 \bar{G}_0}{L_0}$$

$$U_\partial = \frac{G_0}{L_0}$$

Finally, flatness equations for matrix factorizations

- Taking all together, we get "relative bulk-boundary" diffeqs. which play the role of the Gauss-Manin eqs familiar from standard bulk mirror symmetry:

$$K^{(0)}_{KL}(\nabla_t\Psi_a,\Phi_b) = K^{(0)}_{KL}(\partial_t\Psi_a,\Phi_b)+K^{(1)}_{KL}(\Psi_a,\gamma\cdot\Phi_b)-\frac{1}{2}K^{(0)}_{KL}(\sum_i\frac{d_i\phi}{d_iW};\Psi_a,\Phi_b)$$
$$\stackrel{!}{=}\ 0$$

These supposedly determine the proper flat boundary changing representatives ψ(t) incl. moduli dependent renormalisation factors

When combined with the A_∞ structure, the latter eventually determine the t-moduli dependence of all correlation functions!

Example: elliptic curve T_2

- Simplest 1-dim Calabi-Yau: the cubic curve

complex struct modulus

$$T_2: \quad W(x, z(t)) \equiv \frac{1}{3}({x_1}^3 + {x_2}^3 + {x_3}^3) - z(t) x_1 x_2 x_3 = 0$$

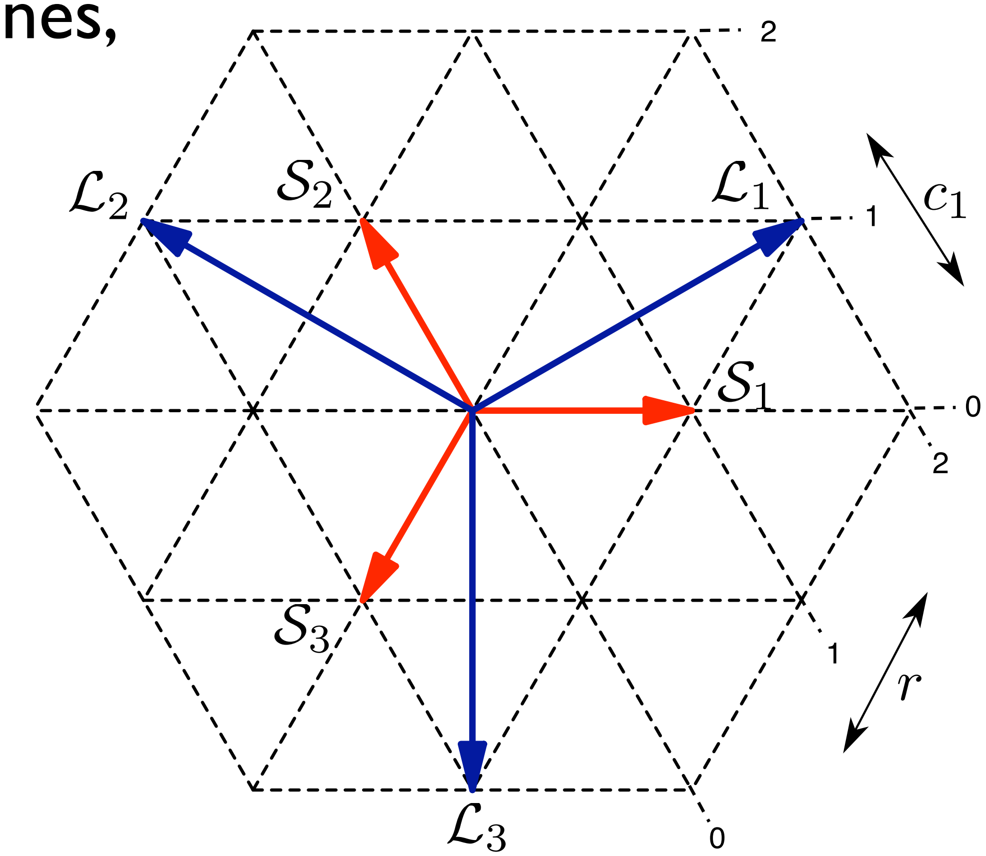
Mirror map: $t(z) = i/\sqrt{3} \frac{{}_2F_1(1/3, 2/3, 1; 1 - 1/z^3)}{{}_2F_1(1/3, 2/3, 1; 1/z^3)}$

- B-type D-branes are composites of D2, D0 branes, characterized by

$$(N_2, N_0; u) \; = \; (\mathrm{rank}(V), c_1(V); u)$$

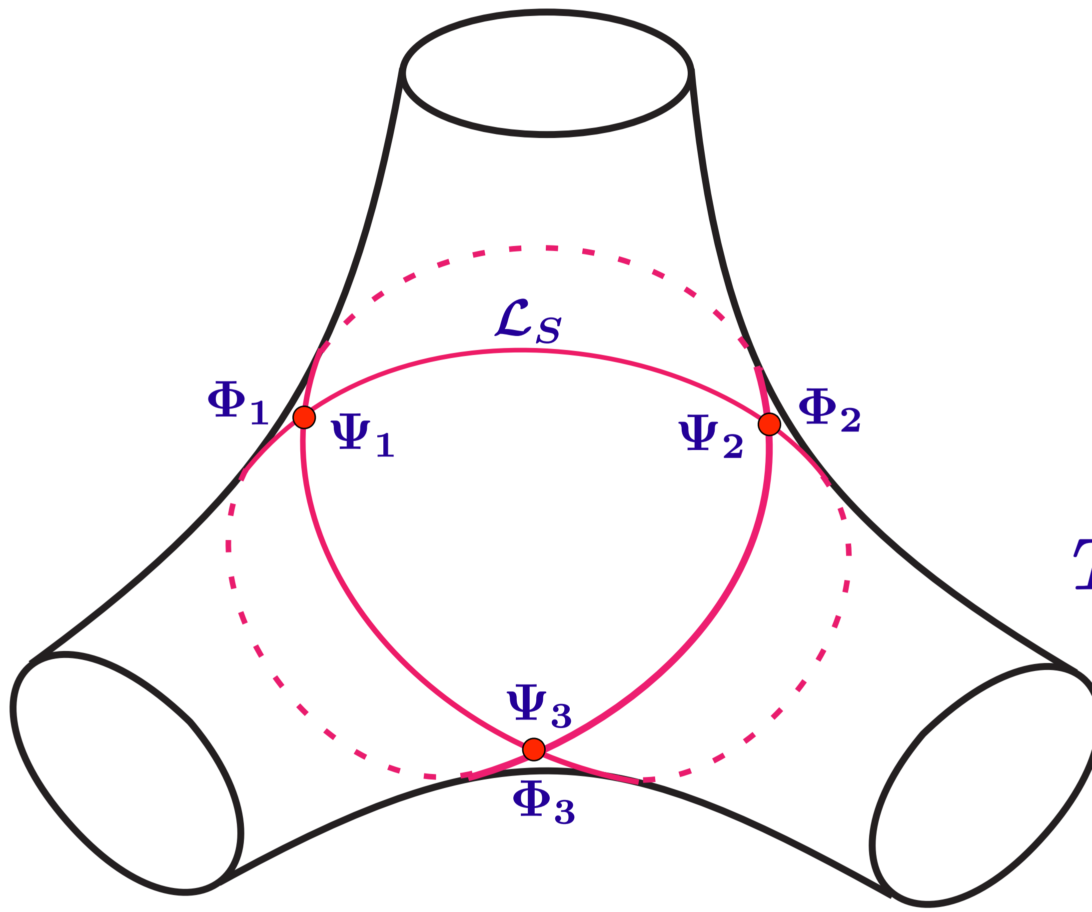
- We will consider the ``long-diagonal" branes with charges

$$(N_2, N_0)_{\mathcal{L}_A} = \{(-1, 0), (-1, 3), (2, -3)\}$$



Seidel lagrangian

- Actually the LG model describes the orbifold T_2/Z_3 (pair of pants), where the 3 branes map into one single, triply self-intersecting brane



$T_2/Z_3 \simeq P_{3,3,3}$

- Need to go to equivariant matrix factorization to describes branes on T_2
  In practice only labels change

Matrix factorization corr to Seidel Lagrangrian

…is given by following 8x8 matrix: $Q = \begin{pmatrix} 0 & p_0 \\ p_1 & 0 \end{pmatrix}$

$$p_0 = \begin{pmatrix} \frac{x_1}{3} & \frac{x_2}{3} & \frac{x_3}{3} & 0 \\ x_2^2 - x_1 x_3 z(t) & x_2 x_3 z(t) - x_1^2 & 0 & \frac{x_3}{3} \\ x_3^2 - x_1 x_2 z(t) & 0 & x_2 x_3 z(t) - x_1^2 & -\frac{x_2}{3} \\ 0 & x_3^2 - x_1 x_2 z(t) & x_1 x_3 z(t) - x_2^2 & \frac{x_1}{3} \end{pmatrix}$$

$$p_1 = \begin{pmatrix} x_1^2 - x_2 x_3 z(t) & \frac{x_2}{3} & \frac{x_3}{3} & 0 \\ x_2^2 - x_1 x_3 z(t) & -\frac{x_1}{3} & 0 & \frac{x_3}{3} \\ x_3^2 - x_1 x_2 z(t) & 0 & -\frac{x_1}{3} & -\frac{x_2}{3} \\ 0 & x_3^2 - x_1 x_2 z(t) & x_1 x_3 z(t) - x_2^2 & x_1^2 - x_2 x_3 z(t) \end{pmatrix}$$

… which satisfies

$$Q^2 \; = \; W(x, z(t))\, 1$$

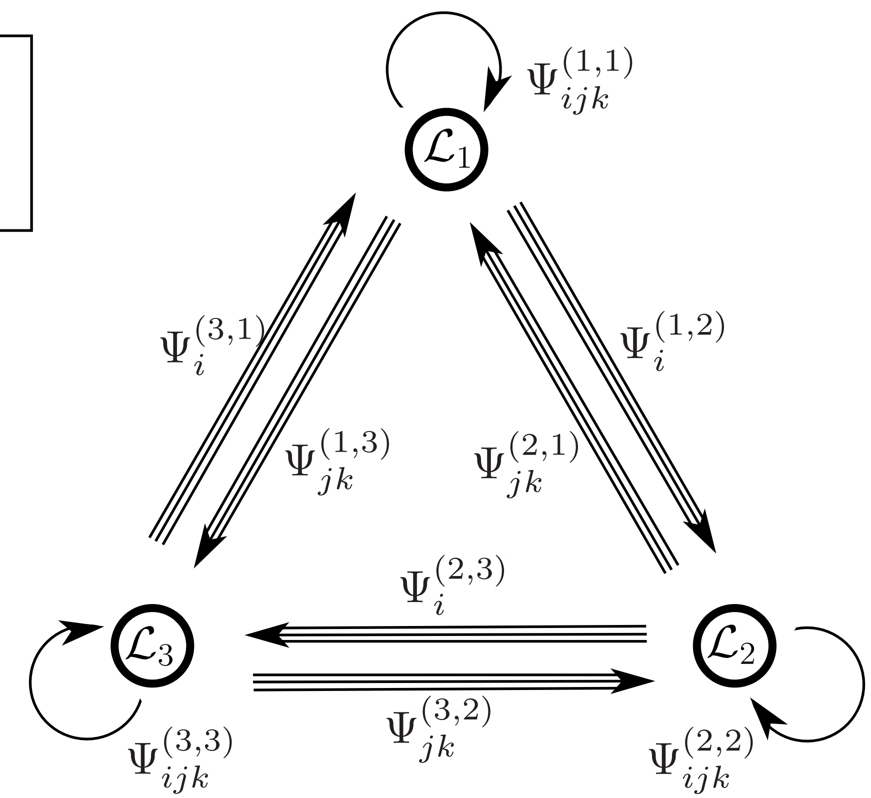
Open string BRST cohomology

- Solving for the BRST cohomology yields explicit moduli dependent matrix valued morphisms, eg.

$$\Psi_1^{(A,A+1)} = g(t)\begin{pmatrix} 0 & q_0 \\ q_1 & 0 \end{pmatrix}$$

$$q_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{3}{2}x_3 z(t) & 3x_1 & 0 & 0 \\ \frac{3}{2}x_2 z(t) & 0 & 3x_1 & 0 \\ 0 & \frac{3}{2}x_2 z(t) & -\frac{3}{2}x_3 z(t) & 1 \end{pmatrix}$$

$$q_1 = \begin{pmatrix} -3x_1 & 0 & 0 & 0 \\ \frac{3}{2}x_3 z(t) & -1 & 0 & 0 \\ \frac{3}{2}x_2 z(t) & 0 & -1 & 0 \\ 0 & \frac{3}{2}x_2 z(t) & -\frac{3}{2}x_3 z(t) & -3x_1 \end{pmatrix}$$



$$[Q, \Psi_a^{(*,*)}] = 0$$

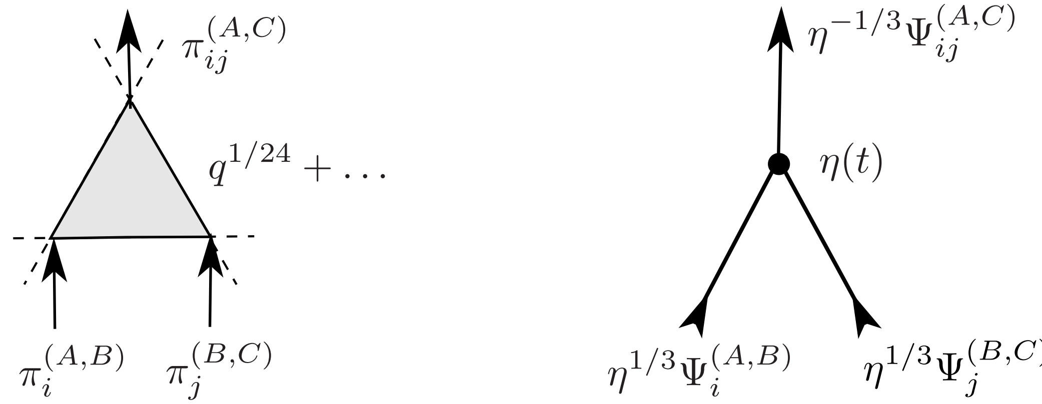
- Again, the issue is to determine the flattening, moduli dependent renormalisation factor g(t)

Solving the previous "relative bulk-boundary" diffeqs yields

$$g(t) = \eta(q)^{1/3}, \quad q = e^{2\pi i t}$$

A-model instantons

- This defines via open string mirror symmetry a quantum Fukaya product m_2:



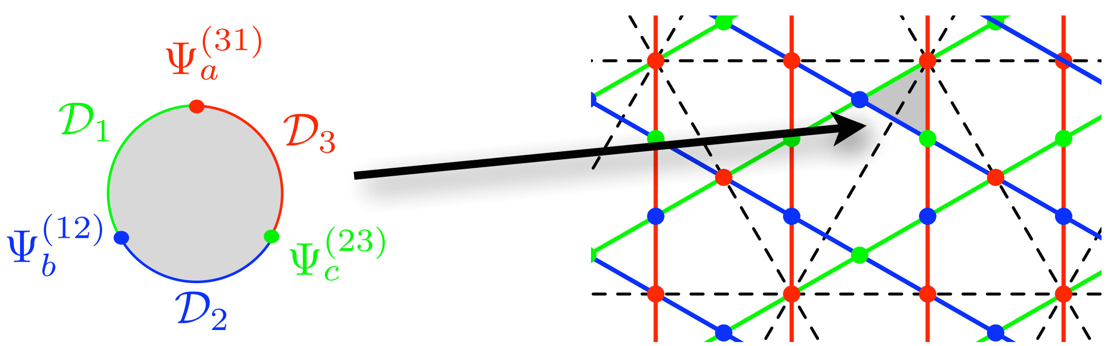
A-Model   B-Model

Functional complexity is entirely due to renormalization factor g(t)!

- Phys. interpretation in top. A-model: 3 point function counts instantons

$$C_{abc}(t) \;=\; \langle\langle \Psi_a^{(1,2)}, m_2(\Psi_b^{(2,3)}\Psi_c^{(3,1)})\rangle\rangle = \epsilon_{abc}\,\eta(q)$$

$$\eta(q) \equiv q^{1/24}\prod_{n>0}(1-q^n)$$



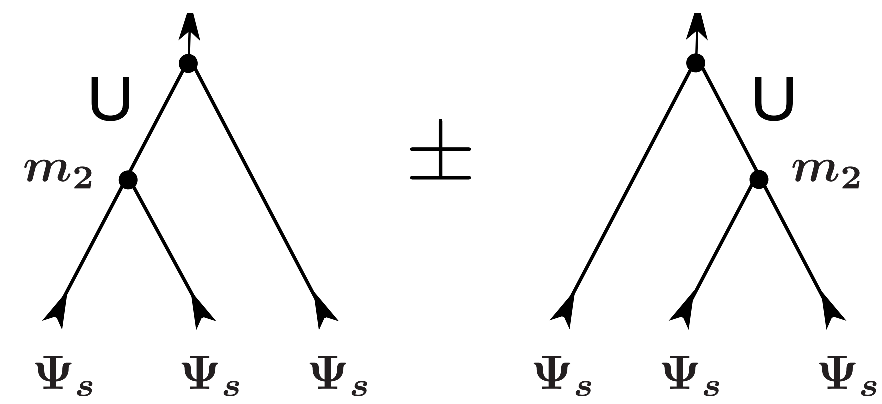
minimal area:
1/24 of fundamental domain

Further B-model correlators

- Define boundary chain

$$\Psi_s = -1/3 \sum s_i \Psi_i$$

Compute m3 via nested trees and propagators



$$m_3(\Psi_s, \Psi_s, \Psi_s) = \frac{\eta(t)}{\zeta(t)} W(s,t) 1$$

$$\zeta(t) = \sqrt{\frac{z'(t)}{z^3(t) - 1}}$$

Weakly obstructed deformation, as expected

Matches results on the A-model side   Cho, Hong, Lau…

Summary and Outlook

- math: Cat of matrix factorizations $\longleftrightarrow$ D(Coh(M))

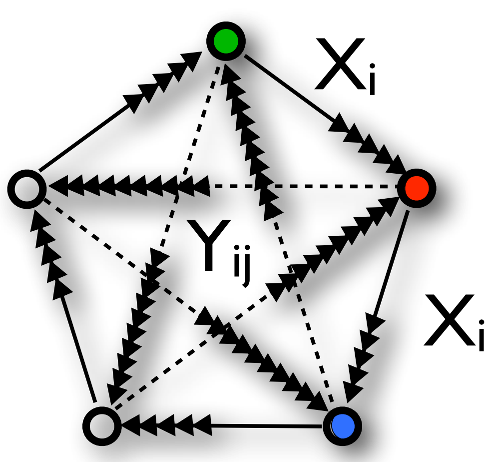
  phys: Boundary B-type TCFT $\longleftrightarrow$ B-type D-branes

- Field theoretical LG model allows to explicitly compute non-trivial correlation functions also for intersecting branes

- Main issue: find suitable Gauss-Manin type differential eqs that determine the proper flat operator bases

  Main tool: matrix analogs for higher residue pairings

- Generalization to M = CY 3-folds, eg. for quintic:



$$\mathcal{W}_{eff} = C_{XXY}(t)\,\mathrm{Tr}XXY + C_{XXYXXY}(t)\,\mathrm{Tr}(XXY)^2 + \ldots$$

t... Kähler modulus, interpolates between Gepner-point (BCFT) and geometrical phase

... infinitely many new results in enumerative geometry