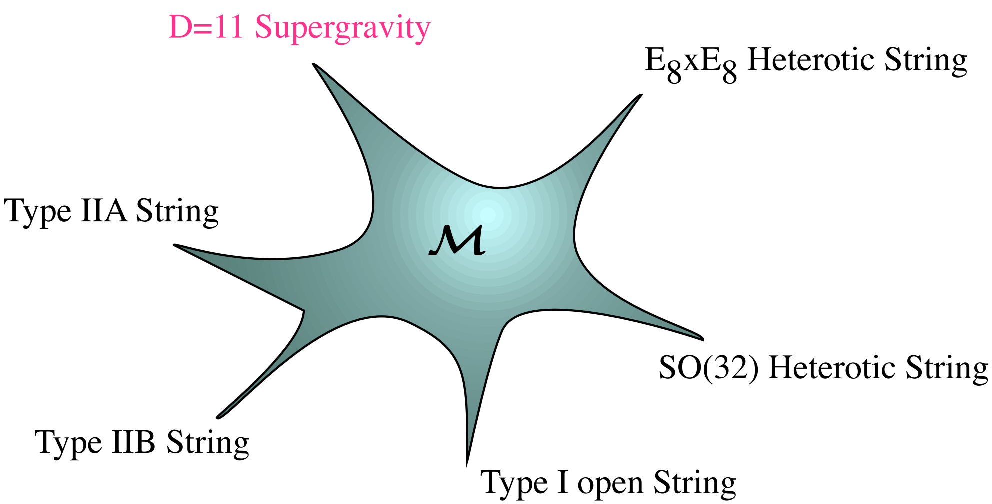
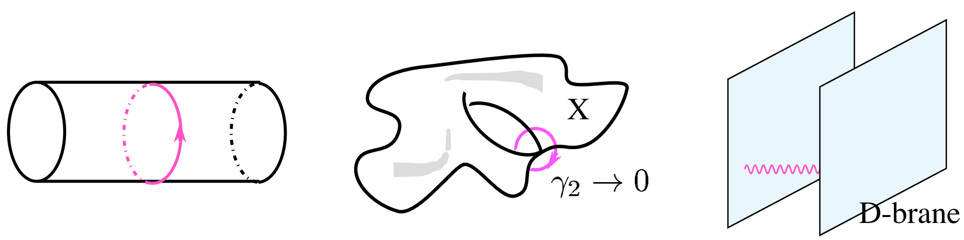
Stringy Geometry and Nonperturbative D-Brane Physics

W.L. 2003



- In stringy geometry, geometrical notions are in general ambiguous ...

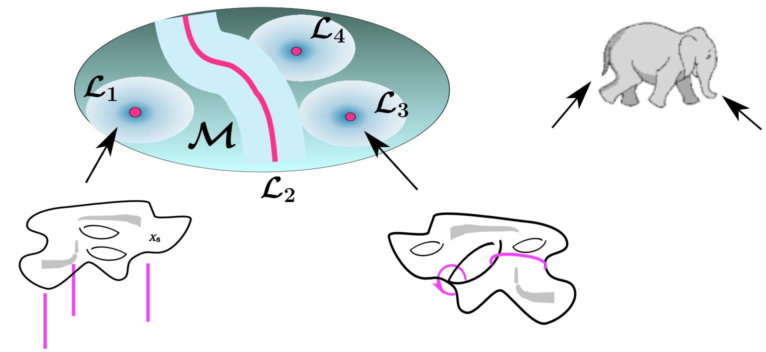
  One and the same theory may have many different dual geometric interpretations. For example gauge theory:



i) heterotic   ii) type II   iii) type I

Moduli Space = Space of Vacuum States, VEV's

- Lagrangian description makes sense only in "**local coordinate patches**" covering the parameter space $\mathcal{M}$:



These describe **different local approximations** of the same theory in terms of **different weakly coupled physical degrees** of freedom.

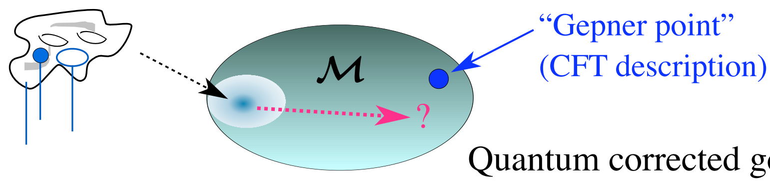
The perturbative physics (local QFT) may look very different in the various local patches (eg, different gauge groups)

- As a general rule, there is no global description that would be valid throughout the whole parameter space;
  no particular theory is more fundamental than the other ones.

Concept of "fundamental degrees of freedom" is ambiguous, at least...

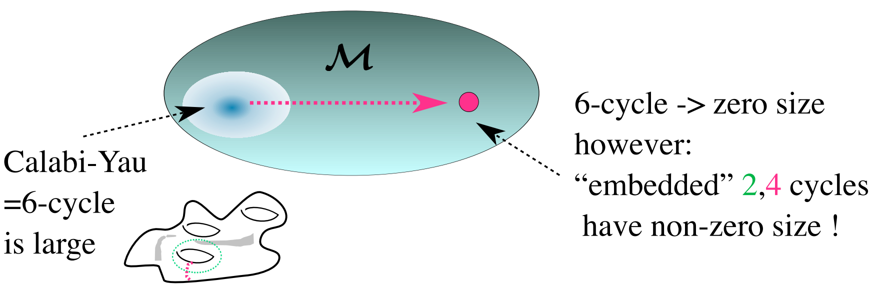
Quantum Moduli Space of D-Branes

- Classical geometry ("branes wrapping p-cycles") makes sense only at weak coupling or large radius:

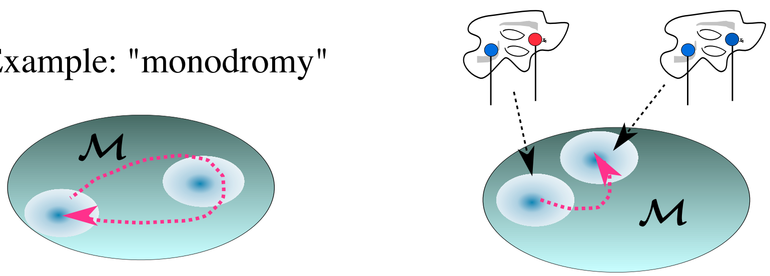


Quantum corrected geometry: (instanton) corrections wipe out concepts of classical geometry

- Example: "quantum volume"



- Example: "monodromy"



Looping around singularieties returns "totally different" brane configuration

A brane-anti brane pair with apparent SUSY breaking, turns into a SUSY preserving brane-brane pair

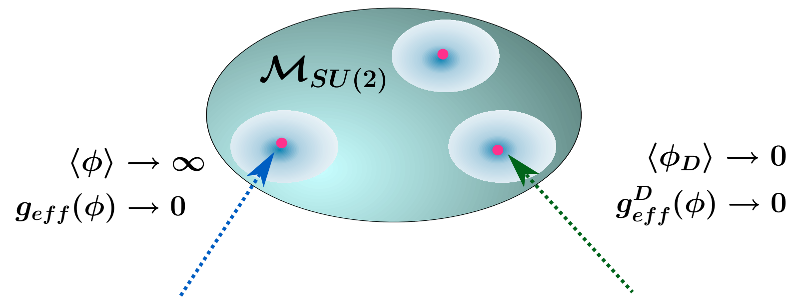
- To meaningfully address such questions, we need to have **global analytical control** of (at least part of) the spectrum and the effective action, like eg. the superpotential.

N=2 SUSY Gauge Theory

- Moduli (parameter) space $\mathcal{M} = \mathcal{M}(u)$, $u \sim < Tr\phi^2 >$
  ϕ ...complex adjoint Higgs field

Seiberg-Witten: Effective gauge coupling gets renormalized, and depends on the Higgs VEV:

$$\tau_{eff}(\phi) = \frac{1}{2\pi}\theta_{eff}(\phi) + 2\pi i \frac{1}{g^2_{eff}(\phi)}$$



gauge fields weakly coupled; monopoles strongly coupled

$$\tau(\phi) = \frac{i}{\pi}\log\left[\frac{\phi^2}{\Lambda^2}\right] - \frac{i}{\pi}\sum_{\ell=1}^{\infty} c_\ell \left(\frac{\Lambda}{\phi}\right)^{4\ell}$$

1-loop — instanton corr

SU(2) gauge theory with instanton corrections

gauge fields strongly coupled; massless monopoles weakly coupled, look like electrons:

$$\tau_D(\phi_D) = \frac{-1}{2\pi}\log\left[\frac{\phi_D^2}{\Lambda^2}\right] - \frac{1}{\pi}\sum_{\ell=1}^{\infty} c^D_\ell \left(\frac{\phi_D}{\Lambda}\right)^{2\ell}$$

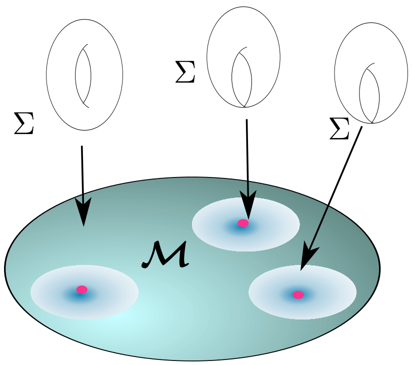
1-loop — non-pert corr

U(1) gauge theory with extra electrons

Resummation of non-perturbative corrections

Quantum Curves and Calabi-Yau Manifolds

- S&W: Interpretation of SYM parameter space as moduli space of an elliptic curve Σ :



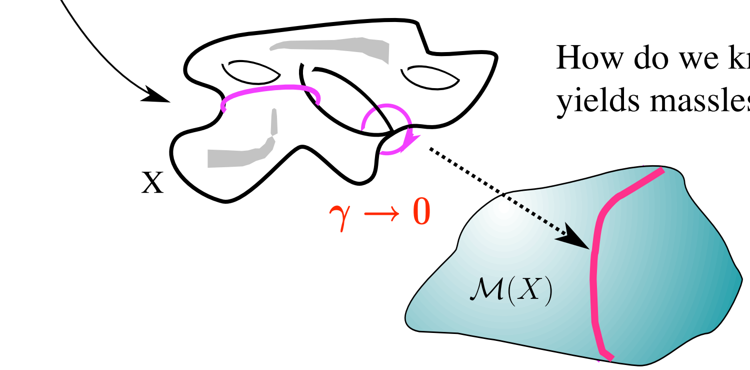
The loci where massless non-perturbatived states appear correspond to singular curves.

What is their meaning ?

- This has a natural interpretation in string theory compactified on some Calabi-Yau manifold X (suitable field theory limit reproduces Σ).

There is a **proliferation of physical degrees of freedom**, obtained from wrapping strings (p=1), membranes (p=2), general p-branes around non-contractible p-cycles of X.

At a given singularity in the parameter space $\mathcal{M}(X)$, a compactification manifold X becomes singular in that some p-dimensional "**vanishing cycle** γ " shrinks to zero size:



How do we know that this yields massless states ?

Supersymmetry and Central Charges

- Supersymmetry allows us do to non-trivial exact computations in toy models, by virtue of its non-renormalization properties that protect many (typically holomorphic) quantities from perturbative corrections.
- .. in particular, quantities related to "BPS"-states:

$$Q_\alpha |BPS\rangle = 0$$

From the algebra of supersymmetry charges

$$\{Q_\alpha, Q_\beta\} = \gamma^\mu_{\alpha\beta} p_\mu + \delta_{\alpha\beta} Z$$

("central charge" Z can be eg. U(1) charge)

and $\langle BPS|\{Q,Q\}|BPS\rangle = 0$

follows for such BPS-states that their mass is exactly given by their charge:

$$m = |Z|$$

- Idea: Find that in semi-classical approximation some state is BPS - this implies it has less degress of freedom than a generic state ("short SUSY multiplet")

But under smooth perturbative and non-perturbative corrections, the number of degrees of freedom cannot jump

→ The state is BPS also in the full quantum theory, and in particular its mass is exactly known !

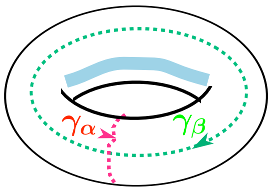
The BPS property is the quintessential basis of our modern non-perturbative techniques.

BPS Geometry: Period Integrals

- Central charges have a topological character.
  - Eg., SU(2) N=2 SUSY gauge theory, the mass of a BPS state is governed by

$$Z(u) \;=\; N\langle\,\phi\,\rangle(u) + M\langle\,\phi_D\,\rangle(u)$$

electric — magnetic charge



$$= \int_\gamma \lambda_\Sigma(u)$$

$$\gamma \;\equiv\; N\,\gamma_\alpha + M\,\gamma_\beta$$

  - Analogously, on a 6d Calabi-Yau space X, the central charge

$$Z(z) \;=\; \int_{\gamma=N_a\gamma_a} \omega_X(z) \;=\; N_a \int_{\gamma_a} \omega_X(z) \;\equiv\; N_a\Pi_a(z)$$

N_a ... el, mag RR charges of wrapped D-brane state

z ... modulus of CY, massless scalar VEV in 4d

$\Pi_a(z)$ ...period integrals, "quantum volumes"

- Significance: periods form the effective lagrangian, ie the holomorphic prepotential $\mathcal{F}$ in N=2 superspace:

$$\mathcal{L}_{eff}(z) \;=\; \int d^4\theta\,\mathcal{F}(z)$$
$$\mathcal{F}(z) \;=\; \Pi_a(z)G^{ab}\Pi_b(z)$$

But what about quantum corrections ?

Type II Strings on Calabi-Yau Manifolds

... give rise to N=2 effective supergravity theory in 4d

- Apart from gravity, there are two decoupled matter sectors:
  - **Vector**-supermultiplet moduli space $\mathcal{M}_V$
  - **Hyper**-supermultiplet moduli space $\mathcal{M}_H$

- This is mirrored in the CY geometry: two sorts of moduli
  - **Kähler** moduli t : size parameters $\sim H^{i,i}_{\bar\partial}(X)$
  - **Complex structure** moduli z: shape parameters $\sim H^{3-i,i}_{\bar\partial}(X)$

  Which Kähler or CS sector corresponds to which vector- or hypermultiplet sector, depends on the specific kind of Type II string theory - more about this later

- There are also two sorts of **supersymmetric p-cycles**, that give rise to BPS particles in 4d when p-branes are wrapped on them:
  - **holomorphic** 0,2,4,6 cycles $\gamma_k^{(2i)} \in H_{2i}(X)$
  - symplectic **special lagrangian** 3-cycles $\{\gamma_\alpha^{(3)}, \gamma_\beta^{(3)}\} \in H_3(X)$

  The Kähler moduli determine the volumes of the holomorphic, and the CS moduli determine the volumes of the SL cycles

- Accordingly there are two sorts of quantum volumes
  - Special lagrangian type:

$$Z(z) \;=\; N_a^{(3)} \int_{\gamma_a^{(3)}} \omega^{(3,0)}(z)$$

  analogous to period integrals over 1-cycles on SW-curves

World-Sheet Instanton Corrections

- holomorphic type:

$$Z(t) \;=\; N_a^{(2i)}\Pi_a(t) \;=\; \int_X Q \wedge e^{\omega^{(1,1)}} + \dots$$

$$Q = Tr[e^F] \wedge \sqrt{\widehat{A}(R)}$$

$$t^k = \int_X (\omega^{(1,1)})^k$$

$$= \; N^{(6)}t^3 + N^{(4)}t^2 + N^{(2)}t + N^{(0)} + \mathcal{O}(e^{-t})$$

= (RR U(1) charges of wrapped brane)*(volumes) + corr.

- In contrast to the SL 3-periods, the classical volumes of 0,2,4,6 cycles do get corrected by world-sheet instantons:

Wrappings of the 1+1d string world-sheet around 2-cycles of the Calabi-Yau X give extra sectors of the path integral with action

$$e^{-S} = e^{-\int_{\gamma(2)} \omega_X^{(1,1)}} = q \equiv e^{-t}$$

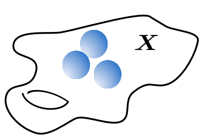
This leads to non-perturbative corrections of the correlation functions, and thus, the eff action:

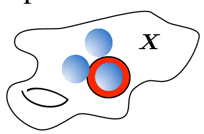
$$\langle\, O_i O_j O_k \,\rangle = \partial_i \partial_j \partial_k \mathcal{F}(t)$$

$$= \; c_{ijk}^{(0)} + \sum_{n_l} d_{n_i n_j n_k} n_i n_j n_k \frac{\prod_m {q_m}^{n_m}}{1 - \prod_m {q_m}^{n_m}}$$

intersection #'s

Integers, count holomorphic maps from 2-spheres into the CY





Mirror Symmetry

- For “every” Calabi-Yau X, there exists a mirror $\widehat{X}$ such that the **Kähler and complex structure** sectors are **exchanged**.

The physical meaning is:

Type IIA strings compactified on X are indistinguishable from Type IIB strings compactified on the mirror $\widehat{X}$

$IIA\,/\,X$	$\longleftrightarrow$	$IIB\,/\,\widehat{X}$

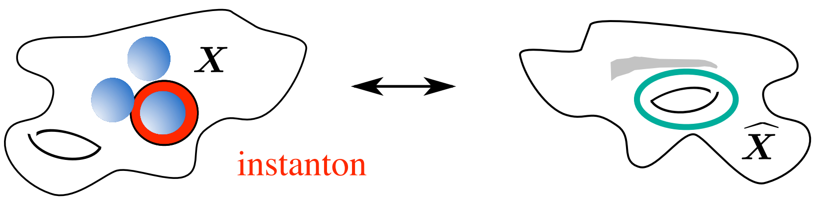
$$\mathcal{M}_H^{\text{(-dilaton)}} \;=\; \mathcal{M}_X^{CS}(z(t')) \;=\; \mathcal{M}_{\widehat{X}}^{KS}(t'(z))$$

$$\mathcal{M}_V \;=\; \mathcal{M}_X^{KS}(t(z')) \;=\; \mathcal{M}_{\widehat{X}}^{CS}(z'(t))$$

(We will consider here only the vector supermultiplet moduli space)

- Mirror symmetry also maps the even, holomorphic cycles into the SL 3-cycles, and v.v.

Type IIA string | Type IIB string



size of 2-cycles governed by Kahler param t_i

size of 3-cycles governed by CS param z_a

$$\Pi_i(t) = \int_{\gamma_i^{(2k)}} (\wedge \omega_X^{(1,1)})^k + \ldots = \int_{\gamma_i^{(3)}} \omega_{\widehat{X}}^{(3,0)} = \Pi_a(z)$$

$$\sim\ t^k + \mathcal{O}(e^{-t}) \qquad\qquad \sim\ (\ln z)^k + \mathcal{O}(z)$$

corrected | exact

- "mirror map": $t = -\ln z + \mathcal{O}(z)$
  thus determines prepotential = effective action:

$$\mathcal{F}(z(t)) = \Pi_a(z(t)) G^{ab} \Pi_b(z(t)) =$$

$$= c\, t^3 + \sum_n d_n Li_3(e^{-nt})$$

integers count maps $S^2 \longrightarrow X$

... provides global analytic control over whole moduli space !

Monodromy of RR Charges

- From mirror symmetry, we have for the central charge $Z = N_a \Pi_a$ of a BPS state:

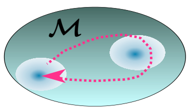
$$N_a^{(3)} \int_{\gamma_a^{(3)}} \omega_{\widehat{X}}^{(3,0)} = N^{(6)} t^3 + N^{(4)} t^2 + N^{(2)} t + N^{(0)} + \mathcal{O}(e^{-t})$$

$$= N^{(6)}(2\mathcal{F} - t\partial t\mathcal{F}) + N^{(4)} \partial_t \mathcal{F} + N^{(2)} t + N^{(0)}$$

3-cycles on $\widehat{X}$ on equal footing $\Longrightarrow$ quantum 0,2,4,6-cycles on X must be on equal footing too !

- Periods are in fact multi-valued sections

  Non-trivial loops in the moduli space $\mathcal{M}_{CS}(\widehat{X})$ will thus induce monodromy:



$$\Pi_a \longrightarrow \Pi_a \cdot R, \qquad R \in Sp(2h^{2,1} + 2, Z)$$

- Consider eg looping around $z \sim e^{2\pi i t} \longrightarrow 0$ in the semi-classical, large volume regime:

$$t = \tfrac{1}{2\pi i} \ln z \longrightarrow t + 1$$

Thus
$$Z = N^{(0)} + N^{(2)} t + \ldots$$
$$\rightarrow \underbrace{(N^{(0)} + N^{(2)} + N^{(4)} + N^{(6)})}_{(N^{(0)})'} + N^{(2)} t + \ldots$$

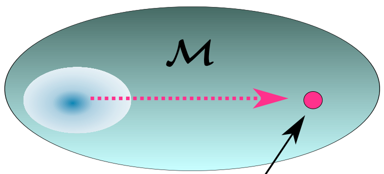
The notion of p-dimensional cycles looses its geometric meaning away from the semi-classical large radius limit !

Quantum Volume

- Recall that massless state in 4d arises if

$$Z = N_A \Pi_A \to 0$$

- Example:
conifold singularity (strong coupling region)



Type IIB: 3-cycle $\gamma_a^{(3)} \to 0$

Type IIA: $$(2F - t\partial t F) \sim t^3 + \dots$$
$$= \int_{\gamma^{(6)}} (\omega_X^{(1,1)})^3 + \dots \to 0$$

$\Longrightarrow$ 6-cycle quantum volume = whole Calabi-Yau X shrinks to nothing!

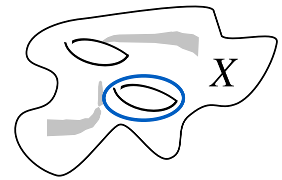
However, the "embedded" 0,2,4 cycles do not have vanishing quantum volumes:

$$(1, t, \partial t \mathcal{F}) \not\to 0$$

The classical geometric picture is completely swamped out by instanton corrections

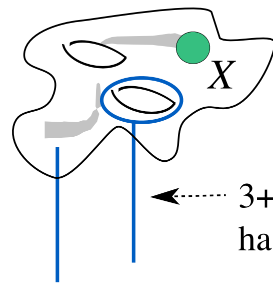
Brane/Flux Configurations with N=1 SUSY

- So far we considered Type II strings on CY threefolds X, which gives N=2 supersymmetry in d=4:



p-branes wrapping supersymmetric p-cycles appear as BPS particle excitations in the N=2 effective theory

We now consider "D-manifolds" with non-compact branes spanning 3+1 dimensions, plus extra fluxes:



3+1d world-volume:
has N=1 SUSY if the brane config is BPS

- This parametrizes a **huge class** of N=1 string geometries!

The effective space time physics will depend non-trivially on the properties of the space X,
and the brane and flux configurations on top of it.

What is the non-perturbative quantum geometry, vacuum states, superpotential, gauge couplings ?

Turning on RR-fluxes

$$\int_{\gamma^{(p)}} H_{RR}^{(p)} = \int_{\gamma^{(p)}} dC_{RR}^{(p-1)} \neq 0$$

- Type IIB strings: p=3

It can be shown that upon turning on $H^{(3)}$ flux, N=2 SUSY is broken to N=1 SUSY with superpotential:

$$\mathcal{W}_{IIB/\widehat{X}} = \int_{\widehat{X}} \omega^{(3,0)} \wedge H_{RR}^{(3)}$$

Denote 3-cycle dual to flux $H^{(3)}$ by $\gamma_H^{(3)}$ and expand in integral symplectic basis of 3-cycles:

$$\gamma_H^{(3)} = N_a \gamma_a^{(3)}, \qquad N_a \in Z \quad \text{(flux numbers)}$$

Then

$$\mathcal{W}_{IIB/\widehat{X}}(z) = \int_{\gamma_H^3} \omega^{(3,0)}(z) = N_a \Pi_a(z)$$

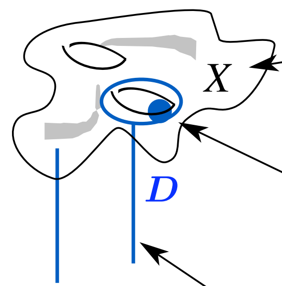
> The superpotential is given a flux-dependent linear combination of period integrals....
>
> It is thus completely determined by the “bulk” closed string geometry:
> spont. broken N=2 SUSY (flux~ aux field)

- Type IIA strings with p=0,2,4,6: mirror to the above

$$\mathcal{W}_{IIA/X}(t) = \int_X \sum_{k=1}^{3} H_{RR}^{(2k)} \left(\wedge\, \omega^{(1,1)}\right)^{3-k} + inst.$$
$$= \mathcal{W}_{IIB/\widehat{X}}(z(t))$$

Moduli of D-brane configurations

- Consider 1/2 BPS configuration D reducing to N=1 SUSY:



CY moduli t, z

brane (and bundle) moduli $\hat{t}, \hat{z}$

4d N=1 SUSY eff action $S_{eff}(t, \hat{t}, z, \hat{z})$ depends how on moduli ?

- Focus on
  - Kähler type moduli:
    - $t \sim \gamma^{(2)}$ sizes of 2-**spheres**
    - $\hat{t} \sim \hat{\gamma}^{(2)}$ sizes of **disks** ending on D-brane D
  - complex structure type moduli:
    - $z \sim \gamma^{(3)}$ sizes of 3-**cycles**
    - $\hat{z} \sim \hat{\gamma}^{(3)}$ sizes of 3-**chains** w. boundary on D

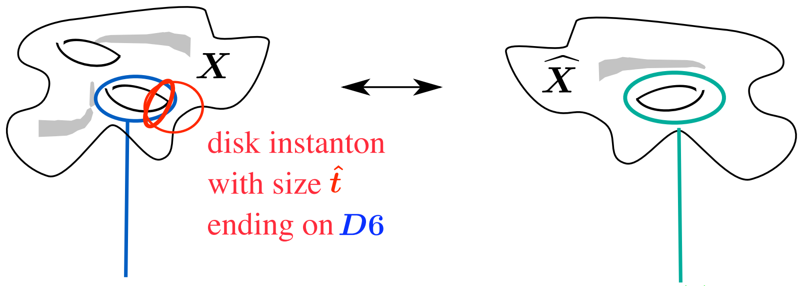
- Decoupling theorems (from boundary CFT):

holomorphic branes	$W(z,\hat{z}),\ \tau(z,\hat{z})$	holom. potentials
	$D(t, t^*, \hat{t}, \hat{t}^*)$	FI D-term potential
SL 3-branes	$W(t,\hat{t}),\ \tau(t,\hat{t})$	holom. potentials
	$D(z, z^*, \hat{z}, \hat{z}^*)$	FI D-term potential

Mirror symmetry and D-Branes

New ingredient: open string (disk) instantons

Type IIA string　　　　　　　　　　Type IIB string



$D6 \simeq \gamma^{(3)} \otimes R^{3,1}$　　　　　$D5 \simeq \gamma^{(2)} \otimes R^{3,1}$

$$\mathcal{W}_{A/IIA}(t,\hat{t}) \;=\; \hat{M}_\ell \hat{\Pi}_\ell \;=\; \mathcal{W}_{B/IIB}(z(t),\hat{z}(t,\hat{t}))$$

$$\sum_{n,m} d_{n,m} Li_2(e^{-nt}e^{-m\hat{t}}) \quad = \quad \hat{M}_\ell \cdot \int_{\hat{\gamma}_\ell^{(3)}(\hat{z})} \omega_{\widehat{X}}^{(3,0)}(z)$$

3-**chain** with boundary

$$\partial\hat{\gamma}^{(3)} = \gamma^{(2)}$$

Complicated corrections due to open and closed string instantons $\longleftrightarrow$ exact result !

Unifying flux and D-brane potentials

- Seek: uniform description of open/closed string backgrounds labeled by the data

$$\left\{X, N_a; \hat{M}_\ell\right\}(t,\hat{t}) \;\cong\; \left\{\widehat{X}, N'_a; \hat{M}'_\ell\right\}(z,\hat{z})$$

closed ; open
string sectors

Recall:
$$\mathcal{W}_{flux} \;=\; N_a \Pi_a$$
$$\mathcal{W}_{D-brane} \;=\; \hat{M}_\ell \hat{\Pi}_\ell$$

Combine:

$$\mathcal{W}_{tot}(z(t),\hat{z}(t,\hat{t})) \;=\; M_\Lambda \Pi_\Lambda \;=\; \int_{\Gamma_\Lambda^{(3)}} \omega_{\widehat{X}}^{(3,0)}$$

$$= N^{(6)} + N_a^{(2)} t_a + N_a^{(4)} \partial_a \mathcal{F} + \hat{M}_k \hat{t}_k + \hat{M}_\ell \mathcal{W}_\ell(t,\hat{t}) + \ldots$$

where $\Gamma_\Lambda^{(3)} \;=\; \{\gamma_a^{(3)}, \hat{\gamma}_\ell^{(3)}\} \in H_3(\widehat{X},\mathcal{D};Z)$

The “relative” homology cycles are closed only up to boundaries lying on the D-brane $\mathcal{D}$.

- The relative homology lattice $H_3(\widehat{X},\mathcal{D};Z)$ is the complete charge lattice of the BPS domain walls in the N=1 theory

The "Special Geometry" of N=1 Vacua

- Basic object: "relative" period vector

$$\Pi_\Lambda = \int_{\Gamma_\Lambda} \omega^{(3,0)} = \{1, t_a, \hat{t}_k, \partial_a \mathcal{F}(t), \mathcal{W}_\ell(t,\hat{t}), \dots\}$$

contains the **holomorphic potentials** of "N=1 Special Geometry"

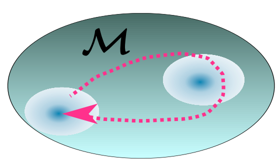
for flux (N=2 closed str) subsector: $\mathcal{W}_a(t) = \partial_a \mathcal{F}(t)$

for boundary (open str) subsector: $\mathcal{W}_\ell(t,\hat{t})$ do not integrate!

...reflects that N=1 SUSY theories are less constrained than their N=2 counterparts

> The Π_Λ can be analytically continued to the strongly coupled, non-perturbative regime, to eg. find exact vacuum states (extrema of the potential).
>
> ..analogous to monopole singularities for N=2 SUSY. Here: tensionless domain walls.

- Monodromy:
  mixes flux and brane numbers
  (note: brane->brane+flux, not v.v)

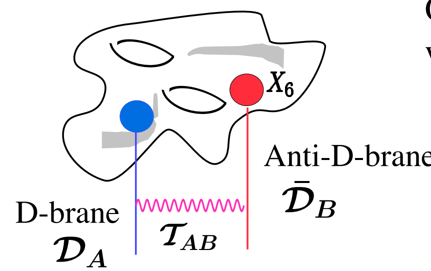


- "Non-renormalization" property:
  boundary (open string) quantities can get modified/corrected by bulk (N=2, closed) string quantities, but not vice versa:

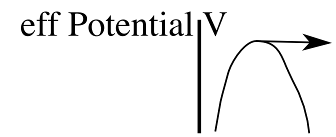
$$z = z(t), \quad \hat{z} = \hat{z}(t,\hat{t})$$

Stability and SUSY Breaking

- In the open string sector, SUSY is typically broken by



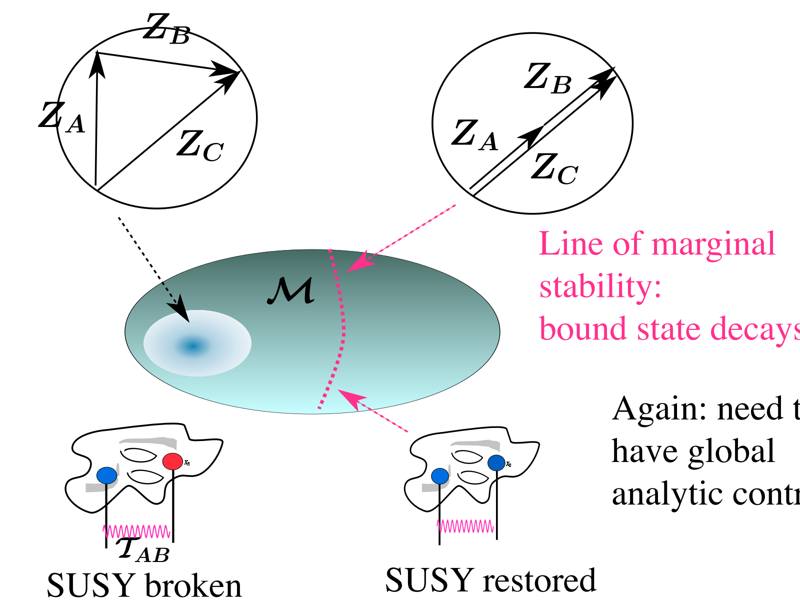
Open string is a tachyon $\mathcal{T}_{AB}$ which implies an unstable vacuum



... equivalent to bound state problem for wrapped branes

- Central charges play a crucial role: $m^2_{AB} \sim Im \ln[Z_A/Z_B]$

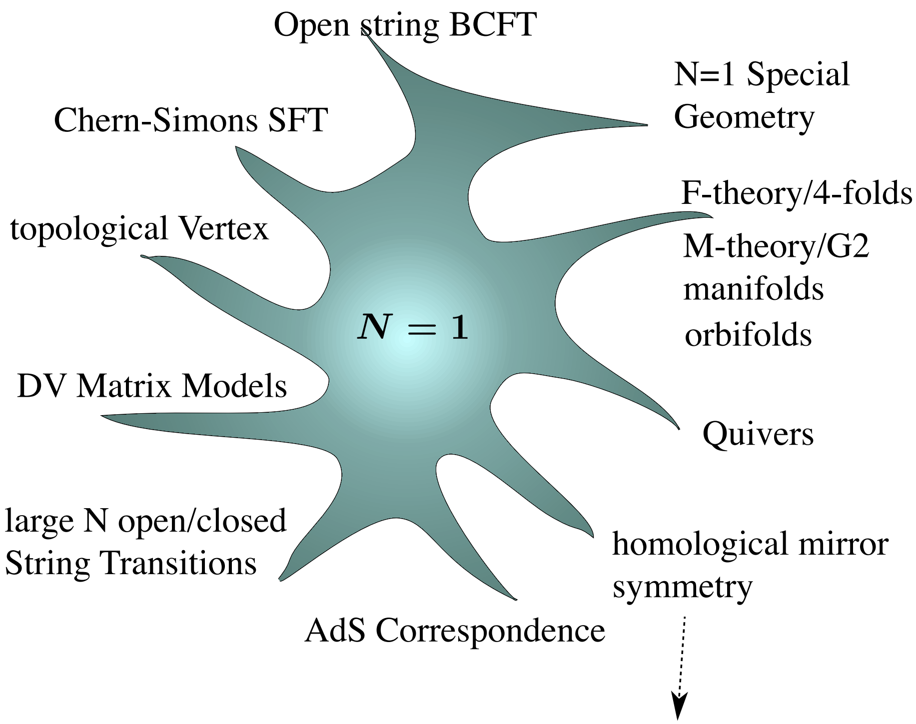
$m_C < m_A + m_B$ $\qquad$ $m_C = m_A + m_B$



Again: need to have global analytic control!

Outlook

We have covered only one “patch” of approaches to/aspects of N=1 supersymmetric string vacua ....



supposed complete, underlying math framework to describe all D-branes, in terms of certain derived categories (Kontsevich)

... equal footing of fundamental branes and their bound states, of branes and anti-branes

$$\mathcal{D}_A \xrightarrow{\mathcal{T}} \mathcal{D}_C \rightarrow \mathcal{D}_B \rightarrow \mathcal{D}_A$$

maps = tachyons   objects=(anti) D-branes