Mirror Symmetry and N=1 Supersymmetry

Part 1 W.Lerche, Trieste Spring School 2003

- Exactly computable quantities are typically "BPS": holomorphic objects protected by SUSY

  N=2: prepotential $\mathcal{F}$ (+ infin sequence $\mathcal{F}_g$)

- Recent progress:

  N=1: superpotential $\mathcal{W}$, gauge coupling τ (+ infin sequence $\mathcal{F}_{g,h}$)

  Non-pert. exact results for string and YM theories !
  (matrix theory, Chern-Simons, mirror symmetry....)

  Reminds of the well-known computation of F for N=2 SUSY: "special geometry", "TFT", "geometric engineering"

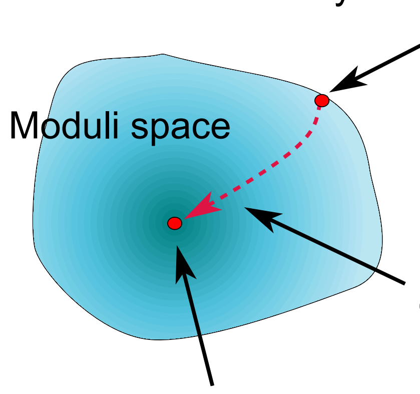
- We will show how to put the computation of N=1 superpotentials on an analogous footing:

  N=1 Special Geometry

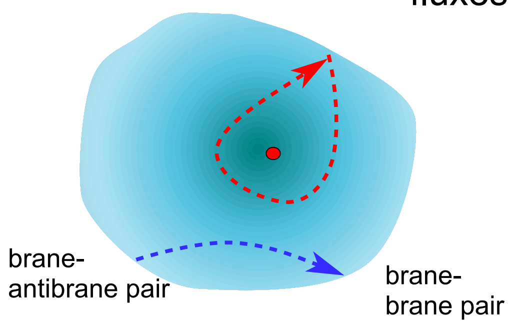
  (main new ingredient: D-branes)

Motivation: Quantum Geometry of D-branes

- Notions of classical geometry (eg., "branes wrapping p-cycles, with gauge bundles on top") make sense only at large radius/weak coupling



- Monodromy:
  brane configuration maps into "different" one involving "other" branes and fluxes



- Stability:
  brane configuration may become unstable

- To address such problems, we need to have full analytical control of F,W, over the full parameter space ....which is more than just a series expansion at weak coupling !

- Note however:

N=2 SUSY: moduli space

N=1 SUSY: W = obstruction to moduli space

STRING THEORY ON CALABI-YAU MANIFOLDS,
By Brian R. Greene, http://arxiv.org/abs/hep-th/9702155

ON THE GEOMETRY BEHIND N=2 SUPERSYMMETRIC EFFECTIVE ACTIONS IN FOUR-DIMENSIONS.
By A. Klemm, http://arxiv.org/abs/hep-th/9705131

Overview

- **Part 1**
  Recap: N=2 special geometry and mirror symmetry
  - Type II strings on Calabi-Yau manifolds
  - Mirror map
  - Topological field theory
  - Hodge variation and DEQ for period integrals
- **Part 2**
  Fluxes and D-branes on Calabi-Yau manifolds
  - Superpotentials from fluxes
  - Mirror symmetry and D-branes
  - Quantum D-geometry
- **Part 3**
  N=1 SUSY and open string mirror symmetry
  - Superpotentials from D-branes
  - Relative cohomology and mixed Hodge variations
  - Differential equations for exact superpotentials

Recap: Type II Strings on Calabi-Yau 3-folds

- For preserving N=2 SUSY in d=4, the compact 6dim manifold X should be Kahler and moreover, a

Calabi-Yau manifold $\left\{ \begin{array}{l} c_1(R) = 0 \\ \text{Holonomy group SU(3)} \\ \text{global holom 3-form } \Omega^{(3,0)} \end{array} \right.$

metric $g_{i\bar{j}} = \partial_i \bar{\partial}_j K$ Kahler potential

Kahler (1,1) form $J^{(1,1)} = i g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}}$

- The string compactification is described by a 2dim N=(2,2) superconformal sigma model on X with c=9, plus a free space-time sector

- The induced N=2 SUSY effective action in d=4 contains massless fields, including hyper- and vector supermultiplets

"decoupling":

Its bosonic sector gives a sigma model with target space

$$\mathcal{M} = \mathcal{M}_V \times \mathcal{M}_H$$

(special Kahler) (quaternionic)

- These massless scalar fields correspond to deformation parameters (moduli) of the CY, X.

These are associated with (p,q) differential forms

$$\omega^{(p,q)} \equiv \omega_{i_1,\dots,i_p,\bar{j}_1,\dots,\bar{j}_q} dz^{i_1} \wedge \dots dz^{i_p} \wedge d\bar{z}^{\bar{j}_1} \wedge \dots d\bar{z}^{\bar{j}_q}$$

which are closed but not exact, ie., are non-trivial elements of the cohomology groups

$$H^{p,q}_{\bar\partial}(X,C) \equiv \frac{\{\omega^{(p,q)} | \bar\partial \omega^{(p,q)} = 0\}}{\{\eta^{(p,q)} | \eta^{(p,q)} = \bar\partial \rho^{(p,q-1)}\}}$$

These give zero modes of Laplacian: $\Delta_{\bar\partial} = \bar\partial \bar\partial^\dagger + \bar\partial^\dagger \bar\partial$ (massless fields in 4d)

- There are two sorts of moduli:

Kahler moduli (size parameters)

$$t_i \sim \omega_i^{(1,1)}, \qquad i = 1, \dots, h^{1,1} \equiv dim H^{1,1}$$

Complex structure moduli (shape parameters)

$$z_a \sim \omega_a^{(2,1)}, \qquad a = 1, \dots, h^{2,1} \equiv dim H^{2,1}$$

($h^{p,q}$ = “Hodge numbers”)

- How do the moduli map to the fields in the effective Lagrangian ?

Mirror Symmetry of CY threefolds

- For “every” Calabi-Yau X, there exists a mirror $\widehat{X}$ such that the Kahler and complex structure sectors are exchanged:

$$H^{1,1}(X) \cong H^{2,1}(\widehat{X})$$
$$H^{2,1}(X) \cong H^{1,1}(\widehat{X})$$

i.e., $h^{p,q}(X) = h^{3-p,q}(\widehat{X})$

- The physical meaning is:

Type IIA strings compactified on X are indistinguishable from Type II strings compactified on the mirror of $\widehat{X}$

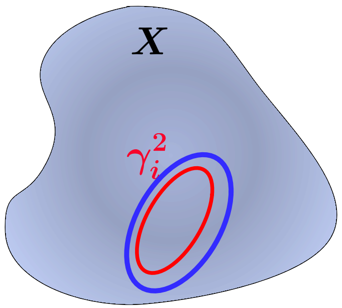
$$\begin{array}{cccccc} & & IIA\,/\,X & \longleftrightarrow & IIB\,/\,\widehat{X} \\ \hline \mathcal{M}_H^{\text{(-dilaton)}} & = & \mathcal{M}_{CS}(X) & = & \mathcal{M}_{KS}(\widehat{X}) \\ \mathcal{M}_V & = & \mathcal{M}_{KS}(X) & = & \mathcal{M}_{CS}(\widehat{X}) \end{array}$$

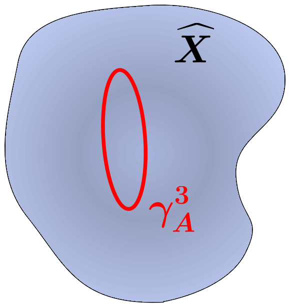
(We will consider here only the vector supermultiplet moduli space)

Why is mirror symmetry useful ?

... basic idea:

Type IIA $\longleftrightarrow$ Type IIB





size of 2-cycles γ_i^2 governed by Kahler param t_i

size of 3-cycles γ_A^3 governed by CS param z_a

Important quantities: quantum volumes ("periods") Π_A

$$\int_{\gamma^{2k}} (\wedge J^{(1,1)})^k + \ldots \quad = \quad \Pi_A \quad = \quad \int_{\gamma^3} \Omega^{(3,0)}$$

$$\sim \; t^k + \mathcal{O}(e^{-t}) \qquad\qquad \sim \; \ln(z)^k + \mathcal{O}(z)$$

world sheet instantons wrapping γ_i^2

no instantons can wrap !

"A-model": corrected

"B-model": exact !

- Significance:

The periods are the building blocks of the prepotential.

Pick an integral basis of homology 3-cycles with

intersection metric $\Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $Sp(2h^{2,1}+2, Z)$ structure

Thus one can split: $\{\gamma_A\} \;\rightarrow\; \{\gamma_a, \gamma_b\}$

and write:

$$\Pi_A(z) \;=\; \left(X_a, \mathcal{F}^b\right) \;\equiv\; \left(\int_{\gamma_a^3} \Omega^{(3,0)}, \int_{\gamma_b^3} \Omega^{(3,0)}\right)(z)$$

In terms of these ''symplectic sections", one has for the prepotential:

$$\boxed{\mathcal{F}(z) \;=\; \frac{1}{2} X_a \mathcal{F}^a(z)}$$

What remains to do is to insert the mirror map:

$$t_i(z) \;=\; -\ln(z_a) + \ldots \;\rightarrow\; z_a = q_i(1+\mathcal{O}(q))$$

$$(q \equiv e^{-t})$$

which gives:

$$\boxed{\mathcal{F}(t) \;=\; \underbrace{\frac{1}{3!} c^0_{ijk} t^i t^j t^k}_{\text{classical}} + \sum_{n_1 \ldots n_r} N_{n_1 \ldots n_r} \underbrace{Li_3({q_1}^{n_1} \ldots {q_r}^{n_r})}_{\text{instanton corrections}}}$$

Integers counting maps $P^1 \rightarrow X$

$$Li_s(q) \equiv \sum_k \frac{q^k}{k^s}$$

Special Geometry of the N=2 Vector-Moduli space

The prepotential F can be understood from three inter-related viewpoints:

A) **as 4d N=2 space-time effective Lagrangian of vector supermultiplets**

gauge couplings $\tau_{ij}(t) = \partial_i \partial_j \mathcal{F}(t)$

"Yukawa" couplings $c_{ijk}(t) = \partial_i \partial_j \partial_k \mathcal{F}(t)$

Kahler potential $K(t, \bar{t}) = -\ln[\bar{X}_a \mathcal{F}^a - X_a \bar{\mathcal{F}}^a]$

B) **2d world-sheet topological field theory**

F = generating function of TFT correlators

$$c_{ijk}(t) \equiv \langle O_i O_j O_k \rangle \;=\; \partial_i \partial_j \partial_k \mathcal{F}(t)$$

OPE: $\boxed{O_i \cdot O_j \;=\; \sum_k c_{ij}{}^k(t) O_k \quad \text{“chiral ring” } \mathcal{R}}$

chiral, primary chiral fields:

$$G^{+}_{-1/2} O_i |0\rangle_{NS} \;=\; G^{\pm}_{+1/2} O_i |0\rangle_{NS} \;=\; 0$$

From N=2 algebra follows:

$$\{G^{+}_{-1/2}, G^{-}_{1/2}\} O_i |0\rangle_{NS} \;=\; (2L_0 - J_0) O_i |0\rangle_{NS} \;=\; 0$$

Thus: $h(O_i) = 1/2 |q(O_i)|$ (no pole in OPE)

- However: need consider pairing of left-, right-moving sectors .... (c,c) and (a,c) rings

Topological Sigma-Model on Calabi-Yau Manifold

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2 z \, [1/2 g_{mn} \partial X^m \bar{\partial} X^n + \\ + i\, g_{\bar{i}j} \lambda^{\bar{i}} D_z \lambda^j + i\, g_{\bar{i}j} \psi^{\bar{i}} D_{\bar{z}} \psi^j + R_{i\bar{i}j\bar{j}} \psi^i \psi^{\bar{i}} \lambda^j \lambda^{\bar{j}}$$

N=(2,2) supercharges:

$$Q_+ \;=\; \oint g_{\bar{i}j} \psi^{\bar{i}} \partial X^j \qquad Q_- \;=\; \oint g_{i\bar{j}} \psi^i \partial X^{\bar{j}}$$

$$\bar{Q}_+ \;=\; \oint g_{\bar{i}j} \lambda^{\bar{i}} \bar{\partial} X^j \qquad \bar{Q}_- \;=\; \oint g_{i\bar{j}} \lambda^i \bar{\partial} X^{\bar{j}}$$

Topological twist:
Redefine spins such that two of these supercharges become scalars to serve as BRST operator with

$$Q_{BRST}{}^2 \;=\; 0$$

This condition projects to a finite number of physical states in the TFT

- Idea: the physical spectrum corresponds to the non-trivial cohomology elements on X, via

$$Q_{BRST} \;\leftrightarrow\; d \;=\; \partial + \bar{\partial}$$

Ambiguity in choosing which supercharges correspond to $\partial,\ \bar{\partial}$!

There are 2 inequivalent possibilities:

$$\boxed{\begin{array}{l} \text{“A-model”: } Q_{BRST} \;=\; Q_+ + \bar{Q}_- \\ \text{“B-model”: } Q_{BRST} \;=\; Q_+ + \bar{Q}_+ \end{array}}$$

A-Model: $Q_{BRST} = Q_+ + \bar{Q}_-$

Observables: $O_A^{(p,q)} = \omega^{(p,q)}_{i_1\ldots i_p \bar{j}_1\ldots\bar{j}_q}\lambda^{i_1}\ldots\lambda^{i_p}\psi^{\bar{j}_1}\ldots\psi^{\bar{j}_q}$

correspond to differential forms on X via:

$$\lambda^i \leftrightarrow dz^i, \quad \psi^{\bar{j}} \leftrightarrow d\bar{z}^{\bar{j}}$$

BRST non-trivial operators $O_A^{(p,q)}$ correspond to cohomology classes $H_{\bar\partial}^{0,q}(\wedge^p T^*) \;\cong\; H_{\bar\partial}^{p,q}(X)$

The Kahler moduli correspond to

$$O_A^{(1,1)} = \omega^{(1,1)}_{i\bar{j}}\lambda^i\psi^{\bar{j}} \in H^{1,1}$$

and generate the (c,c) chiral ring via the OPE:

$$\mathcal{R}^{(c,c)}: \quad O_{A,i}^{(1,1)} \cdot O_{A,j}^{(1,1)} = \sum_k c_{ij}{}^k O_{A,k}^{(2,2)}$$

The 3-point correlators look:

$$c_{ijk}(t) = \langle O_{A,i}^{(1,1)} O_{A,j}^{(1,1)} O_{A,k}^{(-2,-2)} \rangle$$

$$= \int_X \omega_i^{(1,1)} \wedge \omega_j^{(1,1)} \wedge \omega_k^{(1,1)}$$ classical "intersection"

$$+ \sum_{\{u\}} e^{-\int u^* J} \int u^*\omega_i^{(1,1)} \int u^*\omega_j^{(1,1)} \int u^*\omega_k^{(1,1)}$$

Instanton corrections

{u} = holomorphic rational maps $P^1 \rightarrow X$

B-Model: $Q_{BRST} = Q_+ + \bar{Q}_+$

Observables: $O_B^{(p,q)} = \omega^{(p,q)}{}^{i_1\ldots i_p}_{\bar{j}_1\ldots\bar{j}_q}\lambda_{i_1}\ldots\lambda_{i_p}\psi^{\bar{j}_1}\ldots\psi^{\bar{j}_q}$

correspond to differential forms on X via:

$$\lambda_i \equiv g_{i\bar{j}}\lambda^{\bar{j}} \leftrightarrow d/dz^i, \quad \psi^{\bar{j}} \leftrightarrow d\bar{z}^{\bar{j}}$$

BRST non-trivial operators $O_B^{(p,q)}$ correspond to cohomology classes $H_{\bar\partial}^{0,q}(\wedge^p T) \;\cong\; H_{\bar\partial}^{-p,q}(X)$

[Note: a negative degree can be converted to a positive via contraction with the holom 3-form:]

$$\Omega^{(3,0)}: \; \omega^{(-p,q)} \longrightarrow \omega^{(3-p,q)}$$

The complex structure moduli correspond to

$$O_B^{(-1,1)} = \omega^{(-1,1)}{}^{i}_{\bar{j}}\lambda_i\psi^{\bar{j}} \in H^{-1,1} \cong H^{2,1}$$

and generate the (a,c) chiral ring via the OPE:

$$\mathcal{R}^{(a,c)}: \quad O_{B,a}^{(-1,1)} \cdot O_{B,b}^{(-1,1)} = \sum_c c_{ab}{}^c O_{B,c}^{(-2,2)}$$

The 3-point correlators look:

$$c_{abc}(z) = \langle O_{B,a}^{(-1,1)} O_{B,b}^{(-1,1)} O_{B,c}^{(2,-2)} \rangle$$

$$= \int_X (\Omega^{(3,0)}\omega_a^{(-1,1)} \wedge \omega_b^{(-1,1)} \wedge \omega_c^{(-1,1)}) \wedge \Omega^{(3,0)}$$

This is an exact, classical result !
(constant maps only)

Recap: Classical and quantum cohomology rings

- B-Model**:** (complex structure moduli)

(a,c) chiral ring $O_{B,a}^{(-1,1)} \cdot O_{B,b}^{(-1,1)} = \sum_c c_{ab}{}^c\, O_{B,c}^{(-2,2)}$

is isomorphic to the classical cohomology ring

$$H^{2,1}(X) \cup H^{2,1}(X) \;\rightarrow\; H^{1,2}(X)$$

- A-Model**:** (Kahler moduli)

(c,c) chiral ring $O_{A,i}^{(1,1)} \cdot O_{A,j}^{(1,1)} = \sum_k c_{ij}{}^k\, O_{A,k}^{(2,2)}$

is isomorphic to a **quantum deformation** of the cohomology ring

$$H^{1,1}(X) \cup H^{1,1}(X) \;\rightarrow\; H^{2,2}(X)$$

because of the instanton corrections

- Mirror symmetry:

A model on X is equivalent to the B-model on $\widehat{X}$

$$\mathcal{R}^{(c,c)}(X) \;\cong\; \mathcal{R}^{(a,c)}(\widehat{X}) \;\cong\; H^3_{\bar\partial}(\widehat{X})$$

quantum corrected | classical

$$c_{ijk}^{(A)}(t) \;=\; \sum \frac{\partial z_a}{\partial t_i}\frac{\partial z_b}{\partial t_j}\frac{\partial z_c}{\partial t_k}\, c_{abc}^{(B)}(z(t))$$

$$\parallel$$

$$\partial_i\partial_j\partial_k\mathcal{F}(t)$$

C) **Viewpoint of variation of Hodge structures**

Consider in B-model the variation of the holomorphic 3-form under deformations of the complex structure:

$$\begin{aligned}
\Omega^{(3,0)}(z) &\in H^{(3,0)} \\
\delta_z\Omega^{(3,0)}(z) &\in H^{(3,0)} \oplus H^{(2,1)} \\
(\delta_z)^2\Omega^{(3,0)}(z) &\in H^{(3,0)} \oplus H^{(2,1)} \oplus H^{(1,2)} \\
(\delta_z)^3\Omega^{(3,0)}(z) &\in H^{(3,0)} \oplus H^{(2,1)} \oplus H^{(1,2)} \oplus H^{(0,3)}
\end{aligned}$$

(notion of complex structure changes)

Sequence terminates when H^3 is exhausted, so higher derivatives are not independent

Fixing a basis of H^3, we can thus write a matrix DEQ:

(true modulo exact pieces)

$$\nabla_a\varpi \;\equiv\; \Big[\partial_{z_a} - A_a(z)\Big]\cdot\varpi \;=\; 0 \qquad \varpi \equiv \begin{pmatrix}\Omega^{(3,0)}\\ \omega_a^{(2,1)}\\ \omega_a^{(1,2)}\\ \Omega^{(0,3)}\end{pmatrix}$$

Recursive elimination of the higher components gives a set of higher order “**Picard-Fuchs**” operators” acting on integrals of the holom 3-form:

$$\mathcal{L}_a \cdot \int_{\gamma_A^3}\Omega^{(3,0)} \;\equiv\; \mathcal{L}_a\Pi_A \;=\; 0$$

The solutions are thus nothing but the periods we were looking for !

- Flatness of moduli space:

The matrix first oder operator can be decomposed:

$$\nabla_a \;\equiv\; \partial_{z_a} - A_a(z) \;=\; \partial_{z_a} - \Gamma_a - C_a$$

$$\Gamma_a = \begin{pmatrix} * & & & \\ * & * & & \\ * & * & * & \\ * & * & * & * \end{pmatrix} \qquad C_a = \begin{pmatrix} & 1 & & \\ & & (c_a)_{bc} & \\ & & & 1 \\ & & & \end{pmatrix}$$

"Gauss-Manin"-connection — chiral ring structure constants

One can show that $\left[\,\nabla_a,\, \nabla_b\,\right] \;=\; 0$

which means that there are "flat" coordinates, for which the connection vanishes, $\Gamma_a = 0$

These flat coordinates are precisely the Kahler parameter of the associated A-model, $t_i(z_a)$!

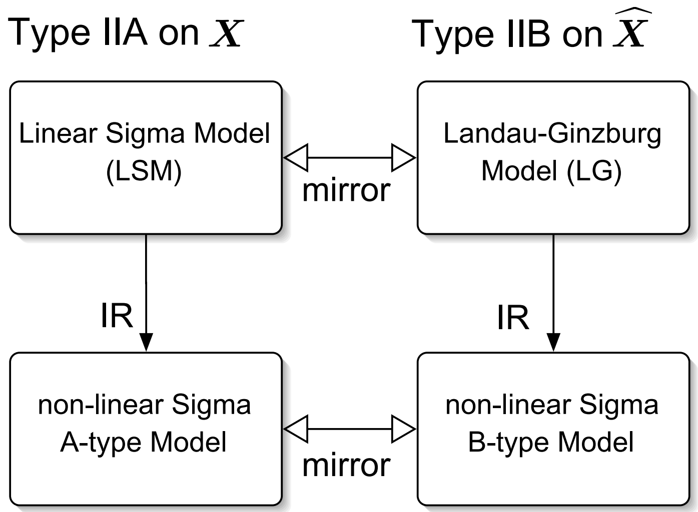
For these coordinates one has:

$$\begin{aligned}\Pi_A(z(t)) &= \left(X_0, X_i, \mathcal{F}^i, \mathcal{F}^0\right)(z(t)) \\ &= \left(1,\, t_i,\, \partial_i\mathcal{F},\, 2\mathcal{F} - t^j\partial_j\mathcal{F}\right) \\ &\sim \left(1,\, t,\, t^2 + \mathcal{O}(e^{-t}),\, t^3 + \mathcal{O}(e^{-t})\right)\end{aligned}$$

so indeed: $\mathcal{F}(t) \;=\; \frac{1}{2}X_a\mathcal{F}^a(z(t))$

Periods and DEQs for toric Calabi-Yau manifolds

Idea: describe 2d superconformal **non-linear** sigma-models as IR limits of a **linear** sigma model (A) or Landau-Ginzburg model (B)



- A-Model on X:

LSM = 2d U(1) gauge theory with fields ϕ_n, charges q_n^i

D-term potential: $V \;=\; D^2,$

$$D \;=\; \sum_n q_n^i|\phi_n|^2 - t_i \;=\; 0$$

Fayet-Iliopoulos parameters = Kahler moduli of X

$(i = 1, ..., h^{1,1}(X))$

The charge vectors q are the most basic data of "toric" Calabi-Yau's X: LSM formulation is canonical

● B-Model on $\widehat{X}$:

Mirror geometry is described by IR limit of a 2d **Landau-Ginzbug** (LG) model, which is defined entirely in terms of the charge vectors q_n^i of the A-model !

LG superpotential: $$W_{LG} = \sum_n a_n y_n$$

with constraint: $$\prod_n y_n{}^{q_n^a} = 1$$

The $\{a_n\}$ parametrize the complex structure deformations of $\widehat{X}$ via

$$\prod_n a_n{}^{q_n^a} = z_a \qquad (a = 1, ..., h^{2,1}(\widehat{X}) \equiv h^{1,1}(X))$$

$$z_a \sim e^{-t_a} + \ldots \quad \text{(mirror map)}$$

● Note: $y_n \in \begin{cases} C & \text{if } \widehat{X} \text{ compact} \\ C^* & \text{if } \widehat{X} \text{ non-compact } (y_n = e^{-\varphi_n}) \end{cases}$

We will consider only non-compact CY in the following

● holomorphic 3-form $$\Omega^{(3,0)}(a(z)) = \prod_n \frac{dy_n}{y_n} e^{-W_{LG}(y,a)}$$

satisfies Picard-Fuchs equation:

$$\boxed{\mathcal{L}_a \, \Omega^{(3,0)} \equiv \left[\prod_{n|q_n^a>0} \left(\frac{\partial}{\partial a_n}\right)^{q_n^a} - \prod_{n|q_n^a<0} \left(\frac{\partial}{\partial a_n}\right)^{q_n^a} \right] \Omega^{(3,0)} = 0}$$

All what remains to do is to change variables a -> z(a)

PF equations immediate once the defining toric data (charge vectors q) of the Calabi-Yau are given !

Example: normal bundle on P^2

● linear sigma model on P^2: $q_n^1 = (1,1,1)$

linear sigma model on $O(-3)P^2$: $q_n^1 = (-3,1,1,1)$

add extra non-compact coo to get CY $c_1 \sim \sum q_n = 0$

● B-model LG potential:

$$W_{LG} = a_0 y_0 + a_1 y_1 + a_2 y_2 + a_3 \frac{y_0{}^3}{y_1 y_2}$$

have used constraint $\dfrac{y_1 y_2 y_3}{y_0{}^3} = 1$

● PF operator: $$\mathcal{L}_1 = \frac{\partial}{\partial a_1}\frac{\partial}{\partial a_2}\frac{\partial}{\partial a_2} - \left(\frac{\partial}{\partial a_0}\right)^3$$

rewriting in terms of $z = \dfrac{a_1 a_2 a_3}{a_0{}^3}$ gives:

$$\mathcal{L}_1(z) = \theta^3 + 3z\theta(1+3\theta)(2+3\theta)$$

...is of generalized hypergeometric type $(\theta \equiv z\partial/\partial z)$

● Solutions for the periods:

$$t(z) \sim \ln(z) + 3\sum (-)^n (3n-1)!(n!)^{-3} z^n$$

$$\partial_t F(z) \sim G_{3,3}^{3,1}(-z||1/3) + G_{3,3}^{3,1}(-z||2/3) \sim \ln(z)^2 + \ldots$$

invert t(z) and insert, integrate:

$$\mathcal{F}(t) = -1/18t^3 + \sum_n N_n Li_3(e^{-nt})$$

indeed integers... counting world-sheet instantons in P^2

Recap: N=2 Special Geometry and Mirror Symmetry

- Quantity of interest: N=2 prepotential of type II compactifications on CY threefolds

$$\mathcal{F}(t) = \frac{1}{2} X_a \mathcal{F}^a(z(t))$$

- Building blocks: periods

$$\Pi_A(z) \equiv \left(X_a, \mathcal{F}^b\right) = \int_{\gamma_A^3} \Omega^{(3,0)}(z)$$

in practice obtained as solution of PF diff eqs;
these are obtained directly from the toric CY data

- (A-model)

$$\partial_i \partial_j \partial_k \mathcal{F}(t) = c_{ijk}(t) =$$

$$= c_{ijk}^{(0)} + \sum_{n_l} N_{n_i n_j n_k} n_i n_j n_k \frac{\prod_m {q_m}^{n_m}}{1 - \prod_m {q_m}^{n_m}}$$

(classical) (instanton corrections)

~ deformed chiral ring structure constants

$$\mathcal{R}^{(c,c)}: \quad O_i \cdot O_j = \sum_k {c_{ij}}^k(t) O_k$$

- Mirror symmetry implies

$$\mathcal{R}^{(c,c)}(X) \cong \mathcal{R}^{(a,c)}(\widehat{X}) \cong H^3_{\bar\partial}(\widehat{X})$$