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Heterotic String-Loop Calculation of the Anomaly Cancelling Term

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Abstract

We calculate the heterotic string one-loop diagram in $2n+2$ dimensions with one external $B_{\mu\nu}$ and n external gauge bosons. We find that it has precisely the right coefficient needed for the Green-Schwarz cancellation mechanism. The calculation is done for the entire set of recently constructed heterotic string theories in 4, 6 and 8 dimensions as well as for the well-known ten-dimensional theories.

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1. Introduction

In 1984 Green and Schwarz [1] made the remarkable discovery that the hexagon anomaly thought to be plaguing the ten-dimensional type I superstring can be cancelled by virtue of a new mechanism. The crucial features of this new mechanism are factorization [2] of the anomaly, that occurs only if the gauge group is SO(32), and the presumed existence in the effective field theory of a term $BTr(F^4)$, together with its gravitational analogues. Here, B is an antisymmetric tensor field that is required to transform non-trivially under gauge transformations [3]. It was quickly realized that the field theory mechanism works also for the gauge group $E_8 \times E_8$ [1] [4]. This led to the construction of the heterotic string [5] which realizes both the above gauge groups.

The direct verification of the absence of gauge and gravitational anomalies in string theory at the one-loop level has been done in [6] for the type I string and in [7] for the ten-dimensional supersymmetric heterotic strings. Furthermore, it has been shown that the factorization of the anomaly for heterotic strings in **any** dimension is a direct consequence of one-loop modular invariance [8]. Still lacking in the understanding of the Green-Schwarz mechanism is how exactly the string provides the anomaly cancelling terms like $BTr(F^4)$. The purpose of this paper is to calculate these terms from string theory.

The question becomes even more interesting in the light of the recent construction of a large number of heterotic string theories in dimensions less than ten, in particular in four dimensions [9] [10] [11]. These are also expected to be anomaly free by means of the Green-Schwarz mechanism. The anomaly cancelling term in $D=2n+2$ dimensions is of the form $BTr(F^n)$. In four dimensions this term becomes $BTr(F)$, which has the additional interesting property that it gives a mass to an "anomalous" U(1) gauge boson.

In this paper we will mainly concentrate on this case, because for a two-point function the analysis is quite easy and instructive. With the insight gained from this calculation, it is then straightforward to compute also the $BTr(F^n)$-term in $2n+2$ dimensions. In 6, 8 and 10 dimensions there are additional terms like $BTr(F^2)Tr(F^{n-2})$, $BTr(R^n)$ etc., which are slightly harder to obtain. For the purpose of this paper, namely to establish the presence of the counter term with the right coefficient, we do not need to consider these terms. However, it remains an interesting problem, to which we hope to come back, to find the relation to the structure of the anomaly as obtained from the character valued partition function [8].

Our calculation provides perhaps the first case where a physically meaningful quantity (ie., the mass of a U(1) gauge boson) is obtained from a one-loop diagram in string theory. It is worth noting that the corresponding field theory diagram, that is a fermion loop with external $B_{\mu\nu}$ and A_ρ legs, is linearly divergent. In string theory, on the other hand, this diagram is finite even though only the same massless fermions and no massive states contribute in the loop. It is proportional to the finite volume of the fundamental domain in modular space (and to $\alpha'^{-1/2}$, providing the correct dimension[1]), giving a factor $2\pi/3$ which will turn out to be exactly what is needed to cancel the anomaly.

[1]We will usually set $\alpha' = 1/2$.

Another reason for our interest in this problem is to demonstrate that for the theories proposed in [11] one can do calculations directly in four dimensions. In this context we like to emphasize that the four dimensional field theory limits are not in general compactifications of field theory limits of ten dimensional strings. (This is true even though the four dimensional strings might be considered, in some generalized sense [12], as compactifications of ten dimensional strings.) Consequently one cannot in general derive the four dimensional BTr(F)-term from the BTr(F^4)-term in ten dimensions by compactification.

In section 2 we review the Green-Schwarz mechanism in field theory with a bit more emphasis on the normalization than has previously been the case. This normalization is of course crucial for the comparison with the string calculation presented in section 3. Although the construction of [11] is based on conformal field theory, it turns out that the NSR formalism is more convenient for this calculation. In particular, we will use the so-called R2-formalism, which has "Feynman rules" closely resembling field theory Feynman rules. This allows us to relate field and string theory normalizations in a straightforward way, as explained in the appendices.

In section 4, we discuss the generalization to 6, 8 and 10 dimensions; the final section contains some concluding remarks.

2. The Green-Schwarz Mechanism

It is well-known that the anomaly in $2n+2$ dimensions due to chiral fermions can be derived from the integrand of the Atiyah-Singer index theorem in $2n+4$ dimensions (see, for instance, [13] [14]). By construction, the theories we are considering can have the following massless fermions [11]:

– – Weyl fermions in some (not necessarily irreducible) representation of the gauge group G and

– – N gravitini plus N Weyl fermions of chirality opposite to that of the gravitini.

The second group of fermions arises as a tensor product of a left-moving bosonic oscillator excitation $\alpha^{i}{}_{(-1)}|0>$ with a right-moving Ramond spinor ground state. In chiral theories N is of course 0 or 1.

The expression generating the anomaly is

$$I(R,F) = \hat{A}(R)\,[Ch(F,r) + N\,Ch(R,v)] \tag{2.1}$$

where $\hat{A}(R)$ is the Dirac genus:

$$\hat{A}(R) = \exp\left[-\sum_{k=1}^{\infty} \frac{B_{2k}}{4k}\frac{1}{(2k)!}\,Tr\left(\frac{iR}{2\pi}\right)^{2k}\right] \tag{2.2}$$

Here, B_{2k} are the Bernoulli numbers and

$$R_{\alpha\beta} = \frac{1}{2} R_{\mu\nu\alpha\beta}\, dx^{\mu} \wedge dx^{\nu} \tag{2.3}$$

is the curvature two-form in the vector representation of the transverse Lorentz group SO(2n). The Chern character is defined as

$$Ch(F,r) = Tr\left[\exp\left(\frac{iF_r}{2\pi}\right)\right] \tag{2.4}$$

where

$$F_r = -\frac{i}{2} g \, F_{\mu\nu}^{a} \, dx^{\mu} \wedge dx^{\nu} \lambda^{a}(r) \tag{2.5}$$

and $\lambda^a(r)$ is the representation matrix of the gauge group G in the representation r. We follow here the conventions of [14], except for the fact that we have expressed F_r in terms of hermitian generators λ^a. The use of the transverse group SO(2n) rather than SO(2n+2) in (2.1) automatically incorporates the ghosts. By r in (2.4) we mean the sum of all irreducible representations of left-handed fermions minus the corresponding sum for right-handed ones.

The anomaly is obtained by applying the descent method to the 2n+4-forms in (2.1)

$$I_{2n+4}(R, F) = d Q^{0}_{2n+3} \tag{2.6}$$

$$\delta Q^{0}_{2n+3} + d Q^{1}_{2n+2} = 0 \tag{2.7}$$

with $\delta A_\mu = \partial_\mu v$. The quantity Q_{2n+2} defined this way is proportional to the anomaly. More precisely, if we define (in Minkowski space)

$$\exp i\Gamma(A) = \int d\psi \, d\bar{\psi} \, \exp i S(\psi, \bar{\psi}, A) \tag{2.8}$$

then the Feynman diagram shown in Fig. 1 is given by

$$\Delta = k_{1}\delta_{1} \int \prod_{i=1}^{n+2} \left[dx_i \exp(i k_i . x_i) \frac{\delta}{\delta A_i} \right] i\Gamma(A) \tag{2.9}$$

where $A_i \equiv A^{a_i}_{\rho_i}(x_i)$ (see Appendix A for conventions). This can be expressed in terms of the gauge variation of $\Gamma(A)$, which can we written in terms of Q_{2n+2}:

$$\Delta = \int \prod_{i=1}^{n+2} \left[dx_i \exp(ik_i x_i) \frac{\delta}{\delta A_i} \right] \left[\delta_v \Gamma(A) \right]_{v=-ig\lambda^{a_i}\exp(ik_i x_i)} \tag{2.10}$$

$$\delta_v \Gamma(A) = -2\pi \int Q'_{2n+2}(v, A) \tag{2.11}$$

The factor -2π is determined by requiring that (2.10) produce the (bose-symmetrized) anomaly calculated with the Feynman rules given in Appendix A.

The gauge and Lorentz transformations of A, ω and B are (cf. Appendix A):

$$\delta A = dv_Y \tag{2.12}$$

$$\delta\omega = dv_L \tag{2.13}$$

$$\delta B = \frac{1}{4g}(\mathrm{tr}\, v_Y\, dA - \mathrm{tr}\, v_L\, d\omega) \tag{2.14}$$

Because of modular invariance [8], all theories we consider have a factorized anomaly, ie.,

$$-2\pi\, I_{2n+4}(R, F) = \mathrm{tr}(R^2 - F^2)\, X_{2n} \tag{2.15}$$

Therefore, as in [2], we find that the anomaly is given by

$$-2\pi \int Q'_{2n+2} = \alpha(\omega'_{2L} - \omega'_{2Y})\, X_{2n} + \beta(\mathrm{tr}\, R^2 - \mathrm{tr}\, F^2)\, X'_{2n-2} \tag{2.16}$$

where

$$\omega'_{2L} = -\mathrm{tr}\, v_L\, d\omega \tag{2.17}$$

$$\omega'_{2Y} = -\,tr\, v_Y\, dA \tag{2.18}$$

and $X^1{}_{2n-2}$ is obtained by the descend method from X_{2n}. α and β are arbitrary parameters with $\alpha+\beta=1$ [15]. The anomaly in (2.15) can be cancelled by the gauge variation of

$$S_c = \int [\beta(\omega_{3L} - \omega_{3Y}) X_{2n-1} \quad - 4g\, B X_{2n}] \tag{2.19}$$

In other words,

$$\delta_v S_c \quad + \delta_v \Gamma(A) \;= 0 \tag{2.20}$$

Note that the BX_{2n} term does not depend on α or β. This is to be expected since it describes a physical quantity.

In the rest of the paper we will be mainly interested in the $Tr(F^n)$-terms in X_{2n} which can be read off from (2.1) by observing that there is only one possible source for a $tr(R^2)Tr(F^n)$-term. By expanding the Dirac genus one gets

$$X_{2n} \;=\; -\frac{1}{48}\,\frac{i^n}{(2\pi)^{n+1}}\,\frac{1}{n!}\,Tr\,F^n + \cdots \tag{2.21}$$

Using the conventions of Appendix A we obtain the following momentum space expression for the $BTr(F^n)$-term:

$$\begin{aligned} &i\Gamma(\zeta_{\mu\nu}\,;\, k_{1\rho_1}, \zeta^{a_1}_{\sigma_1}\,;\ldots;\, k_{n\rho_n}, \zeta^{a_n}_{\sigma_n}) = \\ &\frac{1}{12}\left(\frac{ig}{2\pi}\right)^{n+1}\frac{1}{n!}\,Tr\{\lambda^{a_1}(r)\ldots\lambda^{a_n}(r)\} \\ &\times\, \varepsilon^{\mu\nu\rho_1\sigma_1\ldots\rho_n\sigma_n}\,\zeta_{\mu\nu}\, k_{1\rho_1}\,\zeta^{a_1}_{\sigma_1}\ldots k_{n\rho_n}\,\zeta^{a_n}_{\sigma_n} \end{aligned} \tag{2.22}$$

In order to cancel the anomaly we need to reproduce (2.22) from the string loop calculation. This is demonstrated explicitly in the next two sections.

3. String Loop Calculation in Four Dimensions

The string loop to be calculated is depicted in Fig.2. With the conventions given in Appendix B, this graph can be represented (in the R2-formalism) by the following expression:

$$
\begin{aligned}
i\Gamma(\xi_{\mu\nu}, k_{1\rho}, \xi_\sigma) = \\
-g^2 \mathrm{Tr}\Big[\tfrac{1}{2}\Gamma_* (-2iF_0)\,\Delta\,(\xi_{\mu\nu} P^\mu(0)\,\Gamma^\nu(0)\, e^{ik_0\cdot X(0)}) \\
\times (-2iF_0)\,\Delta\,(\xi_\sigma^I P^I(0)\,\Gamma^\sigma(0)\, e^{ik_1\cdot X(0)})\Big]
\end{aligned}
\tag{3.1}
$$

$(1/2)\Gamma_*$ serves to project onto the periodic-periodic (PP-) sector of fermionic boundary conditions, which is the only sector that describes parity-violating amplitudes proportional to the ϵ-tensor. The sign in (3.1) stems from the fact that we are considering a fermion loop. As explained in Appendix B, F_0, Γ_* and Γ^ν belong to the right-, while P^μ and P^I belong to the left-moving sector. In our case it is sufficient to consider the Cartan-subalgebra generators P^I.

The calculation is greatly simplified by observing that one can only get a non-vanishing contribution by extracting four γ-matrices from the two F_0's and the two Γ^μ's. This automatically restricts the two momentum operators to their zero modes. Performing the γ-trace we find then

$$i\Gamma = -2ig^2 \varepsilon^{\alpha\nu\beta\sigma} Tr'(-1)^F p_\alpha \,\Delta\, P^\mu e^{ik_0\cdot X(0)} \times p_\beta \,\Delta\, P^I e^{ik_1\cdot X(0)} \xi_{\mu\nu} \xi_\sigma^{\,I} \tag{3.2}$$

where Tr′ includes left- and right-moving oscillators, space-time and lattice momenta. Is is easy to see that also P^μ can be restricted to its zero mode because the oscillator part will give rise to terms proportional to $k_0{}^\nu \xi_{\mu\nu} = 0$. It is not hard to see that also P^I contributes only via its zero mode.

Substituting the expressions for the propagators in (B.8) we get

$$\begin{aligned} i\Gamma = ig^2 \varepsilon^{\alpha\nu\beta\sigma} & \int \frac{d^2 z_1}{16\pi |z_1|^2} \int \frac{d^2 z_2}{16\pi |z_2|^2} \\ & \times Tr'\Big[(-1)^F p_\alpha\, z_1^{\,iL_0}\, \bar z_1^{\,i\tilde L_0}\, p^\mu e^{ik_0\cdot X(0)} p_\beta \\ & \times z_2^{\,iL_0}\, \bar z_2^{\,i\tilde L_0}\, p^I e^{ik_1\cdot X(0)} \xi_{\mu\nu} \xi_\sigma^{\,I} \Big] \end{aligned} \tag{3.3}$$

where, of course, $\tilde L_0$ is the Virasoro operator in the Ramond-sector. The fermionic trace is easily evaluated and yields a factor (with $\omega = z_1 z_2$)

$$\prod_{n=1}^{\infty} (1 - \bar\omega^n)^{4-2} \tag{3.4}$$

where the power 4 comes from the Ramond-oscillators and -2 is from the super ghosts. For the bosonic oscillator contribution we find

$$\Big[\prod_{n=1}^{\infty} (1-\bar\omega^n)^{-4-9+2} (1-\omega^n)^{-26+2} \Big] \chi^{\frac{i}{2}k_0\cdot k_1} \tag{3.5}$$

The powers are due to 4 right-moving space-time oscillators, 9 internal oscillators [11], 26 left-moving oscillators and 2 left- and 2 right-moving reparametrization ghosts. The function χ is a Green's function on the torus as defined by Schwarz [16]. Since we will only be interested in the limit where the (external) gauge boson is on-shell (ie., $k_0 \cdot k_1 = -k_1^2 = 0$), we can put $\chi^{k_0 \cdot k_1/2} = 1$.

We now turn to the space-time momentum integral in (3.3). It contributes a factor

$$i \int \frac{d^4 l}{(2\pi)^4} \, l_\alpha \, l^\gamma \, (l + k_1)_\beta \, |z_1|^{\frac{1}{4} l^2} \, |z_2|^{\frac{1}{4}(l+k_1)^2} \tag{3.6}$$

where the factor of i comes from the continuation to Euclidean space. Since this expression appears in (3.3) contracted with an ϵ-tensor it reduces to

$$\frac{i}{4} k_{1\beta} \, \delta_\alpha^\gamma \int \frac{d^4 l}{(2\pi)^4} \, l^2 \exp\left[\frac{1}{4} l^2 \log|\omega|\right] = \frac{-2i}{\pi^2} k_{1\beta} \, \delta_\alpha^\gamma \left(\frac{1}{\log|\omega|}\right)^3 \tag{3.7}$$

by use of symmetric integration.

Finally we discuss the sum over the internal momentum zero modes. For this we switch to the bosonic lattice formulation of the sheet-fermionic degrees of freedom. In the even self-dual lattice formulation of heterotic strings developed in refs. [17] [11] space-time spinors are described by lattice vectors $(w_L, w_R(s))$ which belong to an even self-dual lorentzian lattice $\Gamma_{22;14}$ in the four-dimensional case[1]. The last five components $v_R(s)$ of $w_R(s)$ are spinor weights of the "space-time" lattice D_5: $w_R(s) = (u_R, v_R(s))$; the nine dimensional vector u_R describes internal quantum numbers. The physical massless spinors of given chirality (s) are characterized by $w_L^2 = w_R(s)^2 = 2$, with $v_R(s) = (1/2, 1/2, 1/2, 1/2, 1/2)$. (Note that according to the

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[1] In $2n+2$ dimensions, the relevant lattice is $\Gamma_{24-2n;16-2n}$.

prescription of [11] the last four entries have fixed values).

We now exploit certain specific properties of the PP-sector, which will allow us to evaluate the sum over the internal momentum zero modes without specifying the spectrum explicitly. This is important since we want to cover a large class of theories, which do not even have a direct product structure of left- and right-moving sectors like the well-known supersymmetric ten-dimensional theories.

The PP-sector of the right-moving spinning string has a chiral spinor as its ground state. As was shown in [8], this sector has, by construction [11], the property that the partition function corresponding to the right-movers is a constant with respect to $\bar{\omega}$. This is a consequence of the requirement that all massive excitations of the chiral ground state come in chiral pairs, so that they cancel off in the PP-sector (where spinors and anti-spinors are counted with opposite signs). In other words, the lattice sum cancels the oscillator contributions, that is, the $\bar{\omega}$-dependence of (3.4) and (3.5) so that the integrand in (3.3) is holomorphic in ω (apart from measure factors).

Putting all this together, we find

$$
\begin{aligned}
i\Gamma &= \frac{2}{\pi^2} g^2 \varepsilon^{\mu\nu\beta\sigma} \xi_{\mu\nu} \xi_\sigma^{I} k_{1\beta} \\
&\times \int \frac{d^2 z_1}{16\pi^2 |z_1|^2} \int \frac{d^2 z_2}{16\pi^2 |z_2|^2} \left(\log|\omega|\right)^{-3} \prod_{n=1}^{\infty} (1-\omega^n)^{-24} \\
&\times \sum{}' \langle w_L, w_R | \, \omega^{\frac{1}{2} w_L^2 - 1} p^I \, | w_L, w_R \rangle
\end{aligned}
\tag{3.8}
$$

The primed sum means a sum over all lattice vectors $(w_L, w_R(s)) \in \Gamma_{22;14}$ with $w_R(s)^2 = 2$. (Note that this is not the same as summing over the left-moving part of the lattice $\Gamma_{22;14}$. One would then be neglecting degeneracies due to the components u_R of w_R which are not in D_5). Now we make the standard change of variables [16]

$$\nu = \frac{\log z_1}{2\pi i} \quad ; \quad \tau = \frac{\log z_1 z_2}{2\pi i} = \frac{\log \omega}{2\pi i} \tag{3.9}$$

Then, after performing the ν-integral, (3.8) becomes ($q = \sqrt{\omega} = e^{i\pi\tau}$, and $\eta(q^2)$ is the Dedekind function)

$$i\Gamma = \frac{-g^2}{32\pi^2} \varepsilon^{\mu\nu\beta\sigma} K_{\alpha\beta} \zeta_{\mu\nu} \zeta_\sigma^I \int \frac{d^2\tau}{(\mathrm{Im}\,\tau)^2} \eta(q^2)^{-24} \sum_{m=0}^{\infty} q^{2m} \mathrm{Tr}_m\, p^I \tag{3.10}$$

where the trace is over all lattice vectors obeying $w_L{}^2 = 2m$, $w_R(s)^2 = 2$. Let us now use the fact that the theory was constructed from a self-dual lattice [11] and properties of character valued partition functions [8]. More specifically, the character valued partition function $A(q,F,R=0)$ in question is obtained by "gauging" the lattice function (ν^I is the skew eigenvalue of the field-strength F^I) [8]:

$$A(q,F) = \eta(q^2)^{-24} \sum{}' q^{w_L^2} e^{\nu^I w_L^I} \tag{3.11}$$

It has an expansion in terms of mixed traces in certain basic representations, for which one can take the vector and spinor representations of SO(N). Except for terms proportional to $\mathrm{tr}(F_V{}^2)$, the coefficient $c_k(q^2)$ of a term of order k in F is a modular function of weight $k-n$ (in $2n+2$ dimensions) [8] [11]. In four dimensions we are interested in an expression proportional to TrF, which can be non-vanishing only for a U(1)~SO(2) factor. In the theories of [11] this arises from a D_1 factor of the lattice. Choosing as the basic trace Tr_S, ie., the trace over the D_1 "spinor" representation, it follows that the integrand is proportional to a function $c_1(q^2)$

$$\eta^{-24}(q^2) \sum q^{2m} \mathrm{Tr}_m(p^I) = \frac{\partial}{\partial v_I} A(q,F)\Big|_{v=0} = c_1(q^2)\, \mathrm{Tr}_S(p^I) \tag{3.12}$$

which is thus a modular function of weight zero. This is, of course, exactly what is needed to ensure modular invariance of Γ. Furthermore, since $\mathrm{Tr}_0(p^I)=0$ (the tachyon has no lattice charge), c_1 and hence the integrand is an entire (=non-singular) weight zero modular function and therefore a constant (see, eg., [18]). Accordingly only the massless level can contribute to the integral. (Notice that even if the integrand were not a constant, but a modular function with poles, the τ-integral would have led to the same result). Hence, the integral over modular space gives just the volume of the fundamental domain $\mathcal{F}=\{\tau \mid |\tau|\geq 1, -\tfrac{1}{2}\leq \mathrm{Re}\tau\leq \tfrac{1}{2}, \mathrm{Im}\tau>0\}$

$$\int_{\mathcal{F}} \frac{d^2\tau}{(\mathrm{Im}\,\tau)^2} = \frac{2\pi}{3} \tag{3.13}$$

and we find

$$i\Gamma = -\frac{g^2}{48\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\,k_{1\rho}\,\zeta_\sigma^{I}\,\zeta_{\mu\nu}\;\mathrm{Tr}_1\,p^I\,, \tag{3.14}$$

which is in agreement with the result (2.22) of section 2 since $\mathrm{Tr}_1(p^I)$ is just the trace over the U(1)-charges of the massless fermion states.

4. Generalization to Higher Dimensions

Having already explained the details of the four dimensional calculation, we can here restrict ourselves to a discussion of new aspects appearing in higher dimensions. The generalization of (3.1) is

$$\begin{aligned} & i\Gamma(\zeta_{\mu\nu}\,;\,k_{1\rho_1},\,\zeta_{\sigma_1}^{I_1}\,;\,\ldots;\,k_{n\rho_n},\,\zeta_{\sigma_n}^{I_n}) \\ &= -(-g)^{n+1}\,\mathrm{Tr}\Big[\tfrac{1}{2}\Gamma_*(-2iF_0)\,\Delta\,\big(\zeta_{\mu\nu}\,P^\mu(0)\,\Gamma^\nu(0)\,e^{ik_0.X(0)}\big) \\ &\quad\times\prod_{i=1}^{n}\big\{(-2iF_0)\,\Delta\,\big(\zeta_{\sigma_i}^{I_i}\,p^{I_i}(0)\,\Gamma^{\sigma_i}(0)\,e^{ik_i.X(0)}\big)\big\}\Big] \end{aligned} \tag{4.1}$$

As before, the F_0's and Γ's get restricted to their zero modes. For $P^\mu(0)$ the argument is slightly more subtle than in four dimensions. As in four dimensions, the oscillator contribution from $P^\mu(0)$ will give a linear combination of external momenta. Although $\xi_{\mu\nu}$ contracted with such a linear combination does no longer vanish, it is easy to see that there is no way to saturate the indices on the ϵ-tensor, so that this contribution still vanishes.

In the case of the P^I operators the oscillator contribution does not vanish but gives rise to two-point correlations leading to subtraces of $\mathrm{Tr}(F^n)$. We will not consider these terms here. As before, the zero modes of $P^\mu(0)$ will generate $\mathrm{Tr}(F^n)$, and by analogous arguments one can show that again only the massless fermions contribute.

The generalization of (3.5) in $2n+2$ dimensions is

$$\left[\prod_{n=1}^{\infty} (1-\bar{\omega}^n)^{-12+n} (1-\omega^n)^{-24} \right] \prod_{i<j} (\chi_{ij})^{\frac{1}{2} k_i \cdot k_j} \tag{4.2}$$

(For the definition of χ_{ij} see [16]). In this case the exponents of χ_{ij} are not zero for on-shell external momenta, but we will only consider the leading term in the expansion in external momenta. That is, we keep only terms of the same order in $1/M_{pl}$ as in the field theory anomaly. (Terms of higher order in $1/M_{pl}$ are presumably cancelled by higher order terms in the field theory effective action.) The rest of the calculation is straightforward and yields

$$\begin{aligned} i\Gamma = {} & \frac{1}{12} \left(\frac{ig}{2\pi}\right)^{n+1} \varepsilon^{\mu\nu\rho_1\sigma_1 \cdots \rho_n\sigma_n} \zeta_{\mu\nu} (k_{1\rho_1} \zeta_{\sigma_1}^{I_1}) \ldots (k_{n\rho_n} \zeta_{\sigma_n}^{I_n}) \\ & \times \mathrm{Tr}\, \rho^{I_1} \ldots \rho^{I_n} \end{aligned} \tag{4.3}$$

which again is in agreement with section 2.

5. Discussion

We have shown that the coefficient of the $BTr(F^n)$-term can be calculated from a one-loop string diagram. The result is finite, and agrees with the coefficient expected from anomaly cancellation. This conclusion is expected to hold beyond the already large class of string theories [11] we have explicitly considered, because of the modular properties of the chiral character valued partition function (which plays an important role in our argument) are valid in general.

We want to emphasize that the $BTr(F^n)$-term is neither a "Wess-Zumino term" nor a "local counter term". There is perhaps **some** justification for considering it as a Wess-Zumino term, since just as a Wess-Zumino term it is constructed out of additional bosonic fields and cancels an anomaly. But the similarity stops there; the $BTr(F^n)$-term has a different structure, and therefore it would be incorrect to call it a Wess-Zumino term. Far more important is a possible confusion with "local counter term". Although $BTr(F^n)$ is certainly local and arguably a counter term, by "local counter term" one usually means in this context a term constructed entirely out of gauge fields, which changes different forms of the anomaly into each other. Such a term has no physical meaning, since it depends on how the anomaly was defined in the first place. The terms $\omega_{3L}X_{2n-1}$ and $\omega_{3Y}X_{2n-1}$ in (2.19) are such local counter terms, and indeed their coefficients depend on the parameters α and β appearing in the expression for the anomaly (in diagrammatic language, α and β distribute the total anomaly over the two kinds of gauge bosons that couple to the fermion loop). By choosing $\beta=0$ we can in fact make the local counter terms vanish completely. The coefficient of the $BTr(F^n)$-term on the other hand does not depend on α or β; it is an unambiguous, physical quantity.

A related, potentially confusing issue is the gauge invariance of BTr(F^n). Although in the usual discussion of Green-Schwarz anomaly cancellation one deals with a non-trivial transformation behavior of $B_{\mu\nu}$ (as in (2.14)) as well as of A_μ, the former is only needed to guarantee **off-shell** gauge invariance of the BA^2 vertex obtained from the Chern-Simons terms. Indeed, it is easy to show that the BA^2 vertex (A.15) satisfies the following Ward identity:

$$\begin{aligned} &2i\kappa\, \xi_{\rho\sigma}(r)\, p^\mu \xi_\nu^b(q)\, [q^\mu p^\rho g^{\nu\sigma} + p^\nu q^\rho g^{\mu\sigma}]\, \delta_{ab} \\ &= i\kappa\, \xi_{\rho\sigma}(r)\, \xi_\nu^a(q) [[r^2 - p^2 - q^2] p^\rho g^{\nu\sigma} - 2 p^\nu q^\rho r^\sigma] \end{aligned} \tag{5.1}$$

This vanishes for external legs which are on-shell and have transverse polarizations. Using the $B_{\mu\nu}$-propagator one can construct an A^4 tree graph, which is not gauge invariant; its gauge-variation is cancelled by A^4-terms in $H_{\mu\nu\rho}H^{\mu\nu\rho}$. At the tree-level all the usual Ward-identities are thus satisfied on-shell.

At order $\hbar$ one encounters the BTr(F^n)-term, which by itself satisfies all Ward-identities for the external A_μ-fields (ie., it vanishes upon replacement of $\xi_\mu(k)$ by k_μ) as well as the Ward-identity for the $B_{\mu\nu}$ gauge transformations, $B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\lambda_{\nu]}$. Thus the BTr($F^n$)-term itself causes no problems for gauge invariance of the S-matrix. However, the tree graph constructed out of this vertex and the BA^2 vertex does not satisfy the Ward-identity because of the off-shell terms in (5.1), and it is this gauge variation that cancels the polygon-anomaly (up to local counter terms). The local counter terms on the other hand do not satisfy the Ward-identities, which once more illustrates that they (as opposed to BTr(F^n)) do not have direct physical significance.

The authors of [19] have arrived at different conclusions regarding the $B\mathrm{Tr}(F^n)$-term. Their main interest was however in the four dimensional $\phi F\tilde{F}$ term which arises from the ten-dimensional $B\mathrm{Tr}(F^n)$-term after field-theory compactification (ϕ is an axion, a component of $B_{\mu\nu}$ in the compactified dimensions). Although we have clearly established the existence of the $B\mathrm{Tr}(F^n)$-term, the existence of a $\phi F\tilde{F}$-term depends on the value of an integral over the compactified dimensions, about which we have nothing new to say. Anyway, in most four-dimensional string theories [11] there is no ten-dimensional $B_{\mu\nu}$ field to begin with, and one cannot even define the field ϕ. Nevertheless, some of these theories may have massless gauge-singlet scalars, and for any of them one could calculate the coupling to $F\tilde{F}$ directly in four dimensions.

Finally, the physical relevance of the $B\mathrm{Tr}(F^n)$-term becomes most obvious in four dimensions ($n=1$), where $B_{\mu\nu}$ is equivalent to a scalar, which is eaten by the anomalous photon as in the Higgs-mechanism[1].

Measuring the mass of this photon would in fact be rather interesting, since it would amount to a direct measurement of the volume of modular space. Unfortunately such a striking confirmation of string theory is beyond current experimental dreams.

There has been some discussion of $B_{\mu\nu}-A_\mu$ mixing in another context, namely the open bosonic string [20] [21]. In particular, [21] concludes that this mixing is absent. This is not in disagreement with our results, since we only get such a mixing term in theories with chiral fermions having U(1) charges with a non-vanishing trace.

Acknowledgements

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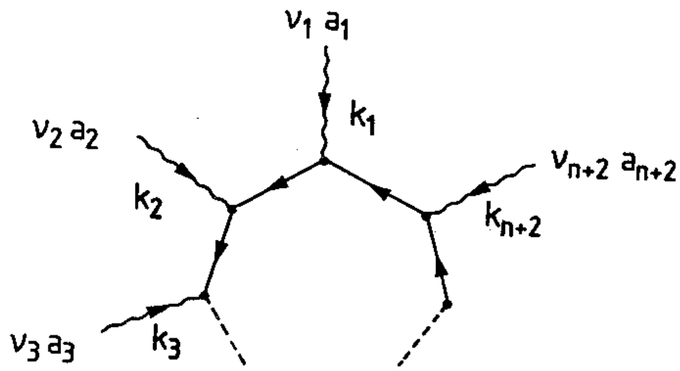

Fig. 1: The polygon anomaly in 2n + 2 dimensions.

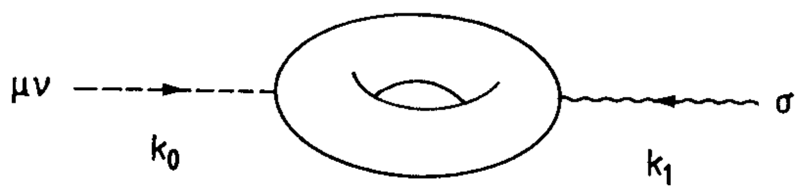

Fig. 2: The BTr(F)-string diagram in four dimensions.

The dashed line represents the $B_{\mu\nu}$ field and the wavy line represents the anomalous photon A_ρ.

APPENDIX A

FIELD THEORY CONVENTIONS

We set up our conventions for field theory, on which the conventions for string theory given in Appendix B will be based. The metric is $\eta_{\mu\nu} = \mathrm{diag}(-+\ldots+)$. The γ-matrices satisfy

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu} \tag{A.1}$$

In $2n+2$ dimensions we define a hermitian matrix

$$\gamma_* = (-i)^{n+2} \gamma^0 \cdots \gamma^{2n+1} \tag{A.2}$$

which satisfies $(\gamma_*)^2 = 1$. The ϵ-tensor is defined by $\epsilon^{0,1,\ldots,2n+1} = 1$, so that

$$\mathrm{Tr}\, \gamma_* \gamma^{\mu_1} \cdots \gamma^{\mu_{2n+2}} = 2^{n+1} i^{n+2} \epsilon^{\mu_1 \cdots \mu_{2n+2}} \tag{A.3}$$

Gauge fields are written in terms of components or as one-forms. They are related as follows:

$$A_\mu = -ig_n \lambda^a A_\mu^a \qquad ; \qquad A = A_\mu dx^\mu \tag{A.4}$$

where g_n is the gauge coupling constant in $2n+2$ dimensions and λ^a is a hermitian generator of the gauge group:

$$[\lambda^a, \lambda^b] = if^{abc} \lambda^c \tag{A.5}$$

The field strength is

$$F_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu] = -ig_n F^a_{\mu\nu}\lambda^a \quad ; \quad \mathcal{D}_\mu = \partial_\mu + A_\mu$$
$$F = \tfrac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu = dA + A^2 \tag{A.6}$$

For the antisymmetric tensor field $B_{\mu\nu}$ we define a two-form

$$B = B_{\mu\nu}\, dx^\mu \wedge dx^\nu \tag{A.7}$$

With similar definitions for the gravitational fields we get the following expression for its field strength:

$$\begin{aligned} H &= H_{\mu\nu\rho}\, dx^\mu \wedge dx^\nu \wedge dx^\rho \\ &= dB - \frac{\kappa}{2g_n^2}\, \mathrm{tr}\left(AF - \tfrac{1}{3}A^3\right) \\ &\qquad + \frac{\kappa}{2g_n^2}\, \mathrm{tr}\left(\omega R - \tfrac{1}{3}\omega^3\right) \end{aligned} \tag{A.8}$$

Here, κ is the gravitational coupling constant, and "tr" indicates a trace over the vector representation of an SO(N) algebra. The bosonic part of the action is given by

$$\begin{aligned} S &= \int dx^{2n+2}\, \mathcal{L} \\ \mathcal{L}_B &= -\frac{1}{2\kappa^2} R - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{3}{2} H_{\mu\nu\rho} H^{\mu\nu\rho} \end{aligned} \tag{A.9}$$

where we have ignored dilaton terms. For Weyl-fermions we get

$$\mathcal{L}_F = -\bar{\psi}\gamma^\mu \mathcal{D}_\mu \psi \qquad (\bar{\psi} = \psi^+ i\gamma_0) \tag{A.10}$$

The generating functionals for disconnected and for connected graphs are given by

$$W(J) = \exp iZ(J) = \int \mathcal{D}\phi\, \exp\left[iS(\phi) + i\int J(x)\phi(x)\, dx^{2n+2}\right] \tag{A.11}$$

where ϕ stand generically for all fields. Vertices in momentum space are defined as follows:

$$i\Gamma_0^k(p_1\cdots p_k)\quad(2\pi)^{2n+2}\,\delta^{2n+2}(\Sigma p_i)$$

$$= \int \prod_{i=1}^{k}\left(d^{2n+2}x_i\,\exp(ip_i\cdot x_i)\,\frac{\delta}{\delta\phi(x_i)}\right)(iS) \tag{A.12}$$

Here p_i is the ingoing momentum on the i^{th} line. (This implies that ∂_μ becomes ip_μ in momentum space.)

With our conventions we get for example the following Feynman rules:

– – three gauge boson vertex:

$$g_n\,\xi_\mu^a(p)\,\xi_\nu^b(q)\,\xi_\rho^c(r)\,f_{abc}\left[g^{\mu\nu}(q-p)^\rho + g^{\nu\rho}(r-q)^\mu + g^{\rho\mu}(p-r)^\nu\right] \tag{A.13}$$

– – gauge boson-fermion vertex:

$$-g_n\,\bar{u}^i\gamma^\mu u^j\,\lambda^a_{ij}\,\xi^a_\mu(p) \tag{A.14}$$

– – BAA-vertex:

$$2i\kappa\,\xi_{\rho\sigma}(r)\,\xi_\mu^a(p)\,\xi_\nu^b(q)\,\delta_{ab}\left[q^\mu p^\rho g^{\nu\sigma} + p^\nu q^\rho g^{\mu\sigma}\right] \tag{A.15}$$

The propagators are $-i/p^2$ (times a projection operator) for bosons and $-p\cdot\gamma/p^2$ for fermions. Then we obtain the following expression for the anomaly diagram in $2n+2$ dimensions:

$$\Delta = \frac{-2\pi i}{(n+2)!}\left(\frac{ig}{2\pi}\right)^{n+2}\epsilon^{\mu_2\nu_2\cdots\mu_{n+2}\nu_{n+2}}\,k_{2\mu_2}\cdots k_{n+2\,\mu_{n+2}}$$
$$\times\,\mathrm{Tr}\,\lambda^{a_1}\{\lambda^{a_2}\cdots\lambda^{a_{n+2}}\} \tag{A.16}$$

This is the result of the contraction of $k_{1\mu}$ with the first external line of the diagram shown in Fig.1, with bose symmetrization; (A.16) is the same expression as the one defined in (2.9). The trace is to be summed over all $(n+1)!$ permutations of the generators between curly brackets.

APPENDIX B

STRING THEORY CONVENTIONS

We will first consider the closed bosonic string. The two dimensional action is

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma\, dt \sqrt{g}\, \eta_{\mu\nu}\, g^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu \tag{B.1}$$

with $0 \leq \sigma \leq \pi$. The field X^μ has the following mode expansion:

$$X^\mu = q^\mu + 2\alpha' p^\mu t + \frac{i}{2}\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n}\left(\alpha_n^\mu e^{-2in(t+\sigma)} + \tilde{\alpha}_n^\mu e^{-2in(t-\sigma)}\right) \tag{B.2}$$

where $\tilde{\alpha}$ refers to right-movers. We define the following operators

$$\begin{aligned} L_0 &= \tfrac{1}{2}\alpha' p^2 + 2N - 2 \\ \tilde{L}_0 &= \tfrac{1}{2}\alpha' p^2 + 2\tilde{N} - 2 \end{aligned} \tag{B.3}$$

where $N = \sum_{n=1}^{\infty} \alpha_{-n}^\mu \alpha_n^\mu$. Then

$$H = \frac{1}{4\pi\alpha'}\int d\sigma \left[(\partial_0 X)^2 + (\partial_1 X)^2\right] = L_0 + \tilde{L}_0 \tag{B.4}$$

From now on we will set $\alpha' = 1/2$.

It is well-known that string amplitudes are related to correlation functions of vertex operators on the world-sheet. It is crucial for our calculation to make this relation precise. We find that the following correspondence gives the right answer for **tree** graphs

$$\begin{aligned} &G(P_1 \cdots P_n) \\ &= \int_0^\infty \frac{d\rho_3 \cdots d\rho_{n-1}}{2^{n-3}} \langle P_n | \hat{V}(P_{n-1}, t_{n-1}) \ldots \hat{V}(P_2, t_2) | P_1 \rangle \\ &\qquad\qquad + \text{other } t\text{-orderings} \end{aligned} \tag{B.5}$$

where $\rho_i = t_i - t_{i-1}$. The operators $\hat{V}(p_i,t_i)$ are vertex operators for the emission of certain physical states. The left hand side is the connected, amputed Green's function for the same physical states. It consists of the field theoretic contribution generated from iZ(J) plus all string corrections. As explained for example in [5], one can convert this expression to

$$\langle p_n | V(p_{n-1},0,0)\, \Delta_B \,\ldots\, \Delta_B\, V(p_2,0,0) | p_1 \rangle \tag{B.6}$$

where

$$\hat{V}(p,t) = \int_0^\pi \frac{d\sigma}{\pi} V(p,\sigma,t) \tag{B.7}$$

$$\Delta_B = \frac{-i}{16\pi} \int \frac{d^2 z}{|z|^2}\, z^{\frac{1}{2}L_0}\, \bar{z}^{\frac{1}{2}\tilde{L}_0} \tag{B.8}$$

and $\qquad z = e^{-2\tau - 2i\sigma} \; ; \quad \tau = it \quad ; \quad d^2 z = 2\, d\mathrm{Re}\, z\, d\mathrm{Im}\, z$

If only one t-ordering is taken into account, the z integration is over the unit disc $|z| \le 1$. The factors 1/2 in (B.5) have to be added to ensure that Δ_B is $-i/(p^2+m^2)$ in that case. To include all t-orderings one must then extend the integration region over the whole complex plane.

To verify this, one can consider three- and four-point tachyon amplitudes. One can describe the tachyon interactions by

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{6} \lambda \phi^3 \tag{B.9}$$

The three tachyon vertex is then $i\lambda$, which is reproduced by a vertex operator $i\lambda e^{ikX}$, if inserted between two states normalized to one. For the four tachyon amplitude we find then

$$\langle k_4 | (i\lambda e^{ik_3 \cdot X(0)}) \Delta_B (i\lambda e^{ik_2 \cdot X(0)}) | k_1 \rangle$$
$$= \frac{i\lambda^2}{8} \frac{\Gamma(-1+1/8\, s)\,\Gamma(-1+\frac{1}{8}t)\,\Gamma(-1+\frac{1}{8}u)}{\Gamma(2-s/8)\,\Gamma(2-t/8)\,\Gamma(2-u/8)} \tag{B.10}$$

This expression behaves like

$$\frac{i\lambda}{s-8}$$

near the tachyon pole s = 8, and this reproduces the s-channel Feynman diagram.

We will now apply the same procedure to fermionic strings. Consider first the Ramond sector. In the literature (see eg. [22]) two formalisms are used. For our purposes the so-called R2-formalism is most convenient, because it has the closest resemblance to field theory. We will simply normalize the vertices and propagators so that the zero modes have the same Feynman rules as fermions in field theory (cf. Appendix A). For this purpose we define the operators Γ^μ and Γ_* (in the right-moving sector of the heterotic string) as

$$\Gamma^\mu(\bar z) = \gamma^\mu + \sqrt{2}\,\gamma_* \sum_{n\neq 0} d_n^\mu \bar z^{-n}$$
$$\Gamma_* = \gamma_* (-1)^F \tag{B.11}$$

where F is the fermion number operator. Then the operator F_0 is defined as

$$F_0 = \int \frac{d\bar z}{2\pi i \bar z}\, \Gamma^\mu(\bar z)\, \tilde P_\mu(\bar z) \tag{B.12}$$

where $\tilde P^\mu(\bar z) = \frac{1}{2} p^\mu + \sum_{n\neq 0} \tilde\alpha^\mu \bar z^{-n}$. The propagator in this formalism is given by

$$\Delta^{R2} = -2iF_0\, \Delta_R \tag{B.13}$$

where Δ_R is given by (B.8), with L_0 and $\tilde{L}_0$ substituted by the corresponding operators of the 2n+2 dimensional heterotic string, with Ramond oscillators in $\tilde{L}_0$.

To define the $B_{\mu\nu}$ and gauge boson emission vertices used in section 3 and 4 we need the compactified and uncompactified momentum operators in the left-moving sector [5]:

$$P^{\mu}(z) = \frac{1}{2}p^{\mu} + \sum_{n\neq 0} \alpha_n^{\mu} z^{-n}$$

$$P^{I}(z) = p^{I} + \sum_{n\neq 0} \alpha_n^{I} z^{-n} \tag{B.14}$$

The vertex operators for $B_{\mu\nu}$ and Cartan subalgebra gauge bosons are

$$V_B^{R2}(k,\sigma,\tau) = -g\,\zeta_{\mu\nu}\, P^{\mu}(z)\,\Gamma^{\nu}(z)\, e^{ik.X}$$

$$V_A^{R2}(k,\sigma,\tau) = -g\,\zeta_{\nu}^{I}\, P^{I}(z)\,\Gamma^{\nu}(z)\, e^{ik.X} \tag{B.15}$$

The coefficients are chosen so that these operators produce the vertices of the Chapline-Manton action, provided one uses the heterotic string relations

$$\begin{aligned} \kappa &= \tfrac{1}{2} g\,(2\alpha')^{n/2} \\ g_n &= g\,(2\alpha')^{(n-1)/2} \end{aligned} \tag{B.16}$$

One may check that with these rules one produces precisely the same expression for a string of fermion propagators and boson emissions as in field theory, if one makes the restriction to zero modes. Note that the conventional string rules correspond to incoming momenta in field theory.

The vertex operators in the R1 formalism can be defined as

$$V^{R1} = \{-2iF_0\,,\,V^{R2}\} \tag{B.17}$$

Using this "picture changing" relation we can convert a string of vertices and propagators in the R2 formalism to the R1 formalism:

$$\cdots (-2iF_0)\,\Delta_R\, V^{R2}(-2iF_0)\,\Delta_R\, V^{R2}\cdots$$

$$= \cdots \Delta_R\, V^{R1}(-2iF_0)\,\Delta_R\, V^{R2}\cdots - \cdots \Delta_R\, V^{R2}(-2iF_0)^2\Delta_R\, V^{R2} \tag{B.18}$$

Using the identity $F_0{}^2 = \tilde{L}_0$ we can write

$$(-2iF_0)^2\Delta_R = \frac{-i}{2\pi}\int d^2z\, \frac{\partial}{\partial \bar{z}}\, \bar{z}^{\frac{1}{2}\tilde{L}_0}\, z^{\frac{1}{2}L_0-1} \tag{B.19}$$

The integrand is a total derivative, which vanishes if the integrand is sufficiently well behaved at the boundary (this is the closed string version of the "cancelled propagator argument"). Repeating this procedure one can replace all R2-vertices except one by R1-vertices. Furthermore, in the one-loop graph one F_0 remains. Although the R2-formalism is most convenient for our purpose, the R1 vertex operators have the advantage of having a straightforward relation to the Neveu-Schwarz sector. The R1 vertex operators are

$$V_B^{R1}(k,\sigma,\tau) = 4ig\,\zeta_{\mu\nu}\, P^{\mu}(z)\,[\tilde{P}^{\nu}(\bar{z}) - \tfrac{1}{4}k_\rho \Gamma^{\nu}(\bar{z})\Gamma^{\rho}(\bar{z})]\,e^{ik\cdot X} \tag{B.20}$$

$$V_A^{R1}(k,\sigma,\tau) = 4ig\,\zeta_{\nu}^{I}\, P^{I}(z)/[\tilde{P}^{\nu}(\bar{z}) - \tfrac{1}{4}k_\rho \Gamma^{\nu}(\bar{z})\Gamma^{\rho}(\bar{z})]\,e^{ikX} \tag{B.21}$$

This implies the Neveu-Schwarz vertex operators are given by

$$V_B^{NS}(k,\sigma,\tau) = 4ig\,\zeta_{\mu\nu}\, P^{\mu}(z)/[\tilde{P}^{\nu}(\bar{z}) - \tfrac{1}{2}k_\rho \psi^{\nu}(\bar{z})\psi^{\rho}(\bar{z})]\,e^{ik\cdot X} \tag{B.22}$$

$$V_A^{NS}(k,\sigma,\tau) = 4ig\,\zeta_{\nu}^{I}\, P^{I}(z)/[\tilde{P}^{\nu}(\bar{z}) - \tfrac{1}{2}k_\rho \psi^{\nu}(\bar{z})\psi^{\rho}(\bar{z})]\,e^{ik\cdot X} \tag{B.23}$$

where we have simply replaced the Ramond field $(1/\sqrt{2})\Gamma^{\mu}(0)$ by the corresponding Neveu-Schwarz field $\psi^{\mu}(0)$.

Using (B.22) and (B.23) one can calculate the BAA and the AAA couplings, and compare these with those of the field theory given in Appendix A. Gauge invariance relates the AAA coupling to the fermion-gauge boson coupling, and this relation should be exactly reproduced in string theory. This provides a non-trivial check on the relation between string theory and field theory conventions.

Since we are dealing only with bosonic emissions, ghosts play a quite simple role in our calculation. One simply adds to $\tilde{L}_0$ the ghost number operators; their effect is only to modify the partition function to the light-cone form.

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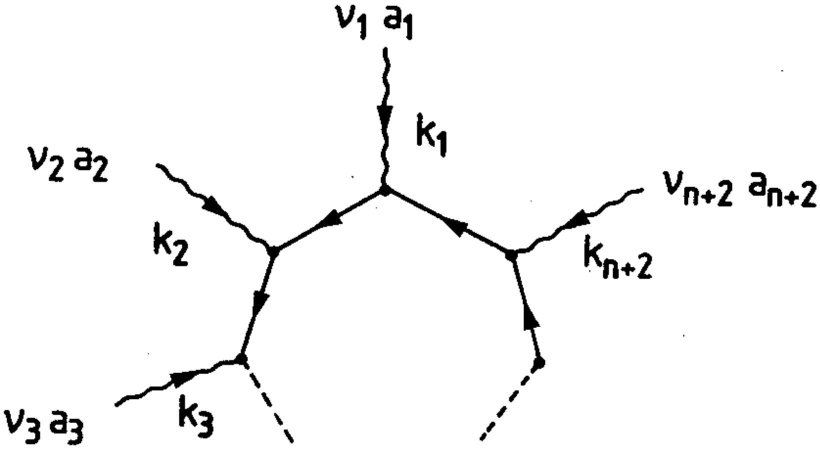

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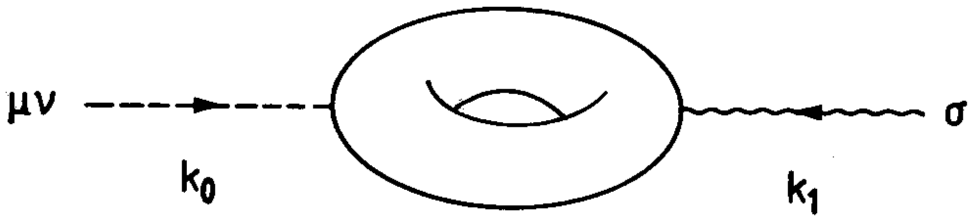

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