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PSEUDO-SYMMETRY CURRENTS AND PCAC IN SUPERSYMMETRIC GOLDSTONE THEORIES

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Abstract

We present a formalism describing dynamical properties of Goldstone fields in N = 1 supersymmetric theories. The concept of pseudo-symmetry currents is introduced. This allows to deal with complex extended pseudo-symmetries of the superpotential in a similar manner as with conventional symmetries. Within this framework, we generalize the usual PCAC relations.

1. Introduction

In the context of composite models, N = 1 supersymmetric Goldstone models have received much attention. The idea is to interpret quarks and leptons as composite "quasi-Goldstone fermions" [1], i.e., as supersymmetric partners of massless Goldstone bosons originating from a spontaneous breakdown of some global internal symmetry group G down to some subgroup H. For some semi-realistic models, see [1,2].

As it turned out, such supersymmetric Goldstone theories have an interesting group theoretical structure [3]. To be more specific, consider a general Lagrangian involving chiral superfields ϕ_i (i = flavor index)

$$\mathcal{L} = \frac{1}{32}\{D^2, \bar{D}^2\} K(\phi_i, \bar{\phi}_i) - \frac{1}{4} D^2 W(\phi_i) - \frac{1}{4} \bar{D}^2 \bar{W}(\bar{\phi}_i) \tag{1.1}$$

Let $\mathcal{L}$ be invariant under transformations of some global flavor group G. In addition, we assume the superpotential $W(\phi_i)$ is such that it leads to a vacuum expectation value (VEV) $\langle\phi_i\rangle \neq 0$ so that G gets spontaneously broken to some subgroup H. The crucial point is that since $W(\phi_i)$ contains only ϕ_i and no $\bar{\phi}_i$, its full symmetry group $\tilde{G}$ is larger or equal to the nonunitary complex extension of G, which is denoted by $\bar{G}$. $\tilde{G}$ gets broken by $\langle\phi_i\rangle$ to $\tilde{H} \supseteq \bar{H} \supset H$ where $\bar{H}$ is the complex extension of H. Now, from the $\tilde{G}$ invariance of $W(\phi_i)$ follows the supersymmetric Goldstone equation [13]

$$m^i_j (T)^k_i \langle\phi_k\rangle = 0 \tag{1.2}$$

where $(T)^k_i$ is a general nonhermitean $\tilde{G}$ generator and m^i_j the chiral superfield mass matrix. It follows that for any broken $\tilde{G}$ generator there exists a massless chiral Goldstone superfield Π. Thus, the number of Goldstone superfields is given by

$$N_\Pi = \frac{1}{2} \dim\left(\tilde{G}/\tilde{H}\right) \tag{1.3}$$

Note that this theorem is independent of the properties of the D-term K in (1.1), even after the introduction of H-gauge fields [4]. Thus, Goldstone physics is completely given by the properties of $W(\phi_i)$. (1.3) suggests that the relevant coset manifold is a (noncompact) Kählerian extension of G/H.

Surprisingly it turned out that N_Π is not fixed for given G and H, in general [1,5]. The reason is that $\tilde{G}$ and $\tilde{H}$ are not fixed for given G and H. Hence, the number of Goldstone superfields depends on the specific linear model. In principle, N_Π can vary between the "minimal" case where $N_\Pi = \frac{1}{2}\dim(G/H)$ [6,F1] over the "fully doubled" case $N_\Pi = \dim(G/H)$ to even higher values [7]. Thus, the total number of massless scalar states is always equal to or greater than the number of Goldstone bosons $N_g = \dim(G/H)$ since the complex scalar component π of a chiral Goldstone superfield Π contains two bosonic degrees of freedom. The interpretation is that the additional scalar states are pseudo-Goldstone bosons, resulting from the breaking of the additional extended pseudo-symmetries $\tilde{G}/G$ of the superpotential. (To define the terminology, we call a symmetry G of the whole Lagrangian (1.1) a true symmetry, $\tilde{G}$ the superpotential symmetry and symmetries of the superpotential which are not symmetries of the whole Lagrangian $\tilde{G}/G$ pseudo-symmetries.) For instance, in the case where $N_\Pi = \dim(G/H)$ (if $\tilde{G} = \bar{G}$, $\tilde{H} = \bar{H}$) every Goldstone boson g is accompanied by a pseudo-Goldstone boson p so that $\pi = g + ip$. Hence the name "fully doubled" case. If $N_\Pi < \dim(G/H)$, which obtains if $\tilde{H} \supset \bar{H}$, one speaks about a "non-fully doubled" case. Here $N_p < N_g$ so that at least some π are of the form $\pi = g + ig$. Of course, the total number of true Goldstone bosons is always $N_g = \dim(G/H)$. Only N_p and therefore $N_\pi = \frac{1}{2}(N_g + N_p)$ depend on the linear theory. What case obtains in a specific model depends on pure group theory or on vacuum expectation values. For example, consider a U(N) invariant theory in which a N-plet $\phi(N)$ has a VEV. Then from (1.2) it follows that we have just N Goldstone superfields:

$$\Pi_a X_a = \left(\begin{array}{c|c} 0 & \begin{matrix} \Pi_1 \\ \vdots \\ \Pi_N \end{matrix} \end{array} \right), \quad \langle \phi(N) \rangle = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ v \end{pmatrix} \tag{1.4}$$

X_a are the broken nonunitary $\bar{G} = GL(N)$ generators. Note that the Goldstone superfields sit in a complex representation of $H = U(N-1)$. However, there are also nondoubled cases (e.g., supersymmetric QCD), where the Goldstone superfields sit in a real representation of H. On the other hand, if one N- and one $\bar{N}$-plet have VEV's, one obtains the fully doubled case

$$\Pi_a X_a = \left(\begin{array}{c|c} 0 & \begin{matrix} \Pi_1 \\ \vdots \end{matrix} \\ \hline \Pi_{2N-1} \cdots & \Pi_N \end{array} \right), \quad \langle \phi(\bar{N}) \rangle = \langle \phi(N) \rangle^T \tag{1.5}$$

where the Π are in a real representation of H. All these issues have been discussed extensively in the literature [3].

In this paper, we want to go one step beyond group theory. In conventional, nonsupersymmetric Goldstone theories, there exists a lot of additional dynamical information like low energy decoupling theorems, reduction formalism involving PCAC and so on. We like to generalize that to supersymmetric theories. For "fully doubled" cases, this has been discussed already in [8,9]. We want to treat the general, i.e., "non-fully doubled" case. The main problem is that in such cases (e.g., (1.4)) the Goldstone spectrum is asymmetric: $(X_\alpha \Pi_\alpha) \neq (X_\alpha \Pi_\alpha)^T$. Hence, we have to find a formulation which takes this feature into account. This is done in the next section where the concept of pseudo-symmetry currents is introduced. These are related only to symmetries of the superpotential which is important since the naive Goldstone theorem (1.2) involves only these symmetries. In Section 3 we prove the field theoretical version of the Goldstone theorem in terms of such pseudo-currents, verifying that they are the relevant quantities. In Section 4 we consider current field identities and discuss the structure of Goldstone field operators. Moreover, in Section 5 Dashen's formula involving only pseudo-symmetry currents is derived. The last section summarizes the paper and indicates interesting points which are not fully discussed in this note.

2. Global Pseudo-Symmetry Currents

In this section, we introduce a more general class of "conserved" quantities, which are related only to the symmetries of the superpotential. We start naively by considering the superfield equations of motion

$$-\frac{1}{4}\bar{D}^2 K_\phi + W_\phi = 0 \tag{2.1}$$

($K_\phi = \partial K/\partial\phi$, $W_\phi = \partial W/\partial\phi$) corresponding to the Lagrangian (1.1). By multiplying (2.1) by

$$\delta_\alpha \phi = i T_\alpha \phi \tag{2.2}$$

we see that for every symmetry of $W(\phi)$

$$\delta_a W = W_\phi \delta_a \phi = 0 \tag{2.3}$$

there exists a quantity J_a which obeys a "conservation law"

$$\bar{D}^2(K_\phi)\delta_a \phi = \bar{D}^2(K_\phi \delta_a \phi) = 0$$

or

$$\bar{D}^2 J_a = 0 \tag{2.4}$$

where

$$J_a = K_\phi T_a \phi \tag{2.5}$$

Here we used the chirality property $\bar{D}_{\dot{\alpha}}\phi = 0$. In the nonsupersymmetric case there is no analogue of that, thus, (2.4) is a specific supersymmetric feature. For canonical kinetic terms $K = \bar{\phi}\phi$, $J_a = \bar{\phi} T_a \phi$.

Now, as remarked above, if $W(\phi)$ is invariant under (unitary) transformations corresponding to some (compact) Lie group G

$$\delta\phi = \lambda_a \delta_a \phi = i \lambda_a T_a \phi \tag{2.6}$$

with real parameters λ_a and hermitean generators T_a, it is also invariant under these transformations with complex λ_a. That is, W is invariant under the non-compact non-unitary complex extension $\bar{G}$. Actually, the full invariance $\tilde{G}$ of W may even be larger than $\bar{G}$ [7]. Hence, from (2.3) it follows that for any such symmetry there exists some "conservation law" (2.4).

There appears a slight complication. In (2.6) the generators T_a are hermitean, suggesting the current for canonical kinetic terms $J_a = \bar{\phi} T_a \phi$ is real

$$J_a = \bar{J}_a \tag{2.7}$$

However, since λ_a are complex, we could equally well take a nonhermitean complex linear combination of the T_a as basis of the Lie algebra of $\tilde{G}$, e.g., the nonhermitean step generators in the case of $G = SU(N)$, $\tilde{G} = SL(N)$. Then (2.7) is not fulfilled. It will turn out later that it depends on the specific problem which choice of basis is suitable. Therefore we had better use a notation which does not specify the basis; just define the general $\tilde{G}$-generator as an unspecified complex linear combination

$$
\begin{aligned}
T_\lambda &= \lambda_a T_a \neq T_\lambda^+ = T_{\lambda^*} \\
\lambda &\in \mathbb{C} , \quad T_a = T_a^+ , \quad \lambda_a^* \lambda_a = 1
\end{aligned}
\tag{2.8}
$$

and similarly

$$
\begin{aligned}
J_\lambda &= \lambda_a J_a \\
\delta_\lambda &= i T_\lambda
\end{aligned}
\tag{2.9}
$$

To understand how the quantity J_λ is related to the usual hermitean conserved current, we repeat our arguments more carefully, employing functional superfield formalism. The global symmetry operator, which determines the variation of the action Γ under a variation of a chiral field ϕ is given by

$$
\hat{\Omega}_\lambda(\phi) = \int d^4x \, \hat{\omega}_\lambda(\phi) = \int dS \, \delta_\lambda \phi \frac{\delta}{\delta \phi}
\tag{2.10}
$$

where $dS = d^4x d^2\theta = d^4x(-1/4 \, D^2)$ is the chiral measure. Acting with $\hat{\omega}_\lambda(\phi)$ on Γ yields basically the equations of motion for ϕ:

$$
\hat{\omega}_\lambda(\phi)\Gamma = \frac{1}{16} D^2 \bar{D}^2 (K_\phi \delta_\lambda \phi) - \frac{1}{4} D^2 W_\phi \delta_\lambda \phi
\tag{2.11}
$$

Consequently, we have as Ward identities for the pseudo-currents J_λ

$$
-\frac{1}{4} D^2 \delta_\lambda W(\phi) + \frac{i}{16} D^2 \bar{D}^2 J_\lambda - \hat{\omega}_\lambda(\phi)\Gamma = 0
\tag{2.12a}
$$

$$
-\frac{1}{4} \bar{D}^2 \delta_{\lambda^*} \bar{W}(\bar{\phi}) - \frac{i}{16} \bar{D}^2 D^2 \bar{J}_{\lambda^*} - \hat{\omega}_{\lambda^*}(\bar{\phi})\Gamma = 0
\tag{2.12b}
$$

Defining the full variation by $\hat{\omega}_{\lambda,\lambda^*} = \hat{\omega}_\lambda(\phi) + \hat{\omega}_{\lambda^*}(\bar{\phi})$ Eqs. (2.12) combine to

$$
-\frac{1}{4} D^2 \delta_\lambda W - \frac{1}{4} \bar{D}^2 \delta_{\lambda^*} \bar{W} + \frac{i}{16} \left(D^2 \bar{D}^2 J_\lambda - \bar{D}^2 D^2 \bar{J}_{\lambda^*} \right) - \hat{\omega}_{\lambda,\lambda^*} \Gamma = 0
\tag{2.13}
$$

Thus, using the equations of motion $\hat{\omega}_{\lambda,\lambda^*}\Gamma = 0$, there is for any symmetry of the superpotential a generalized pseudo-conservation law [F2]

$$D^2\bar{D}^2 J_\lambda - \bar{D}^2 D^2 \bar{J}_{\lambda^*} = 0 \tag{2.14}$$

(In fact, from (2.4) we have the even stronger statement $\bar{D}^2 J_\lambda = 0$.) (2.14) is more general than the usual conservation law. Define hermitean (J_λ^+) and antihermitean (J_λ^-) currents by

$$\begin{aligned} J_\lambda^\pm &= \tfrac{1}{2}\left(J_\lambda \pm \bar{J}_{\lambda^*}\right) \\ &= \tfrac{1}{2}\left(K_\phi T_\lambda \phi \pm \bar{\phi}\, T_{\lambda^*} K_{\bar{\phi}}\right) \end{aligned} \tag{2.15}$$

which represent vector superfields. Then from (2.14) it follows on shell where $\hat{\omega}_{\lambda,\lambda^*}\Gamma = 0$:

$$[D^2, \bar{D}^2]\, J_\lambda^+ + \{D^2, \bar{D}^2\}\, J_\lambda^- = 0 \tag{2.16}$$

or, defining

$$J_{\lambda\mu}^+ = -\tfrac{1}{4}\, \sigma_\mu^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}]\, J_\lambda^+ \tag{2.17}$$

so that

$$\tfrac{i}{16}[D^2, \bar{D}^2]\, J_\lambda^+ = -\partial^\mu J_{\lambda\mu}^+ \tag{2.18}$$

one arrives at

$$\partial^\mu J_{\lambda\mu}^+ - \tfrac{i}{16}\{D^2, \bar{D}^2\}\, J_\lambda^- = 0 \tag{2.19}$$

Clearly, this constitutes a true conservation law only if the second term vanishes. To make contact with the usual current conservation laws, note that the Lagrangian (1.1) can be written in the form

$$\mathcal{L} = -\tfrac{1}{4} D^2 L - \tfrac{1}{4}\bar{D}^2 \bar{L} \tag{2.20}$$

where

$$L = -\tfrac{1}{8}\bar{D}^2 K + W \quad , \quad \bar{D}_{\dot{\alpha}} L = 0 \tag{2.21}$$

Varying $\mathcal{L}$ leads to the conventional identity for hermitean currents

$$\delta_\lambda \mathcal{L} = \partial^\mu J^{conv.}_{\lambda\mu} + \hat{\omega}_\lambda \Gamma , \quad \lambda \in \mathbb{R} \tag{2.22}$$

or

$$-\frac{1}{4} D^2 \delta_\lambda L - \frac{1}{4} \bar{D}^2 \delta_\lambda \bar{L} = \frac{-i}{16} [D^2, \bar{D}^2] J^{conv.}_\lambda + \hat{\omega}_\lambda \Gamma \tag{2.23}$$

where

$$\begin{aligned} J^{conv.}_\lambda &= \frac{1}{2} \left(K_\phi T_\lambda \phi + \bar{\phi} T_\lambda K_{\bar{\phi}} \right) \\ J^{conv.}_{\lambda\mu} &= -\frac{1}{4} \sigma_\mu^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] J^{conv.}_\lambda \end{aligned} \tag{2.24}$$

are the conventional real vector supercurrents. Thus any true symmetry of the whole Lagrangian leads to a true conservation law

$$\partial^\mu J^{conv.}_{\lambda\mu} = 0 \tag{2.25}$$

Comparing (2.23) with (2.13) we see that for real λ $J^+_\lambda = J^{conv.}_\lambda$ and its divergence is related to the variation of the $\tilde{G}$ noninvariant part of $\mathcal{L}$:

$$\frac{i}{16} \{ D^2, \bar{D}^2 \} J^-_\lambda = \frac{1}{32} \{ D^2, \bar{D}^2 \} \delta_{\lambda,\lambda^*} K(\phi, \bar{\phi}) \tag{2.26}$$

However, for canonical kinetic terms $K = \bar{\phi}\phi$ the variation of $\{D^2, \bar{D}^2\} K$ vanishes

$$\{ D^2, \bar{D}^2 \} \bar{\phi} (T_\lambda - T_{\lambda^*}) \phi = 0 \tag{2.27}$$

because of the chirality property $\bar{D}_{\dot{\alpha}} \phi = 0$ and the equations of motion $\bar{D}^2 \bar{\phi} T_\lambda \phi = W_\phi T_\lambda \phi = \delta_\lambda W = 0$, assuming also $\delta_{\lambda^*} W = 0$. This is not surprising since for canonical kinetic terms $J^+_\lambda = \bar{\phi} (T_\lambda + T_{\lambda^*}) \phi$ is a hermitean G-current for which (2.19) should turn into a true conservation law (2.25).

Moreover, for canonical kinetic terms we have a conservation law (2.25) also for the non-unitary $\tilde{G}/G$ currents. This is due to $\bar{J}_\lambda = J_\lambda$ in that case. That is, from $D^2 \bar{D}^2 J_\lambda = 0$ and $D^2 \bar{D}^2 J_{\lambda^*} = 0$ it follows also $\bar{D}^2 D^2 J_\lambda = 0$

by conjugation. Defining $J_{\lambda\mu}$ by (2.17), subtracting $\bar{D}^2 D^2 J_\lambda$ from $D^2\bar{D}^2 J_\lambda$ and using (2.18) one obtains

$$\partial^\mu J_{\lambda\mu} = 0 \,, \qquad \lambda \in \not{c} \tag{2.28}$$

Hence, for canonical kinetic terms there is for any superpotential symmetry a true conservation law.

Imagine now gauging some subgroup S of G. As it is well known, this breaks G explicitly down to some subgroup S' ⊇ S. In other words, the conventional currents $J^{conv.}_{\lambda\mu}$ get a divergence (we set $K = \bar{\Phi}\, e^V \phi$)

$$\partial^\mu J^{conv.}_{\lambda\mu} \equiv -\frac{i}{16}\left[D^2, \bar{D}^2\right] J^{conv.}_\lambda \Big|_{\theta=\bar{\theta}=0} = \frac{i}{32}\left\{D^2,\bar{D}^2\right\} \bar{\Phi}\left[e^V, T_\lambda\right]\phi\Big|_{\theta=\bar{\theta}=0} \tag{2.29}$$

and are modified to

$$J^{conv.}_\lambda = \frac{1}{2}\,\bar{\Phi}\left\{e^V, T_\lambda\right\}\phi \tag{2.30}$$

(V are the S gauge fields). The unbroken global group S' is defined to be generated by those T_λ which obey $[V,T_\lambda] = 0$, so that the corresponding currents are conserved. Thus, the remaining symmetries G/S' have become pseudo-symmetries. However, the psèudo-symmetry conservation law (2.14) remains unspoiled [F3], the only effect of gauging is that the pseudo-currents get changed to

$$J_\lambda = \bar{\phi}\, e^V T_\lambda\, \phi \,, \qquad \bar{J}_{\lambda^*} = \bar{\phi}\, T_{\lambda^*}\, e^V \phi \tag{2.31}$$

This certainly was expected since (2.13) involves only the superpotential symmetry $\tilde{G}$. But that is not changed because gauging affects only the D-term $K(\phi,\bar{\Phi})$. In short, gauging S results in G → S' but $\tilde{G} \to \tilde{G}$.

We can summarize this section as follows: For any superpotential symmetry $\tilde{G}$ there exists a generalized pseudo-conservation law (2.14) which however does not imply a true conservation law (2.25), in general. Only in the case of canonical kinetic terms or (unitary) symmetries of the whole Lagrangian are there true conservation laws and thus conserved charges. Corresponding to non-unitary symmetries $\tilde{G}/G$ (or $\tilde{G}/S'$ in the gauged case) there exist no true conservation laws for non-canonical kinetic terms. However, this does not imply that Eqs. (2.12)-(2.14) have no physical importance. In fact, we claim that at least for Goldstone physics precisely the pseudo-currents J_λ are the relevant quantities, together with the Ward identities (2.12)-(2.13). This is suggested by the fact, as already noted in the introduction, that the Goldstone spectrum depends only on the broken $\tilde{G}$-generators, i.e., on the properties of the superpotential.

3. Goldstone Theorem

In this section we show that the weaker pseudo-conservation law (2.12) is sufficient to establish Goldstone's theorem, indicating the importance of the pseudo-current J_λ . (In Section 6 we will discuss shortly another class of Goldstone theorem.) Consider (2.12a), assuming some superpotential symmetry:

$$\frac{i}{16} D^2 \bar{D}^2 J_\lambda - \hat{\omega}_\lambda(\phi)\,\Gamma = 0 \tag{3.1}$$

Introducing sources for J_λ and ϕ we define the generating functional

$$e^{iW[J_J, J_\phi]} = \int [d\phi]\, e^{i\left\{\Gamma(\phi) + \int dS\, J_\phi \phi + \int d\bar{S}\, \bar{J}_\phi \bar{\phi} + \int dV\, J_J J\right\}} \tag{3.2}$$

Using (3.1) we get as Ward identity for $W[J_J, J_\phi]$

$$\left(\frac{i}{16} D^2(1)\bar{D}^2(1)\frac{\delta}{\delta J_{J_\lambda}(1)} - \hat{\omega}_\lambda\left(J_\phi(1)\right)\right) W[J_J, J_\phi] = 0 \tag{3.3}$$

where

$$\hat{\omega}_\lambda\left(J_\phi(1)\right) = -\frac{1}{4} D^2(1)\,\delta_\lambda\, J_\phi(1)\,\frac{\delta}{\delta J_\phi(1)} \tag{3.4}$$

and (1) denotes some point in superspace. Differentiating (3.3) with respect to J_ϕ (2) yields

$$\left(\frac{i}{16} D^2(1)\bar{D}^2(1)\frac{\delta^2}{\delta J_{J_\lambda}(1)\,\delta J_\phi(2)} + \frac{1}{4} D^2(1)\, T_\lambda\, \delta(1,2)\frac{\delta}{\delta J_\phi(1)}\right) W = 0 \tag{3.5}$$

Here $\delta(1,2)$ is the chiral delta function. Integration over x_1 and writing

VEV's yields finally

$$\frac{1}{16}\int dx_1\, D^2(1)\,\bar{D}^2(1)\,\langle 0|T\{J_\lambda(1)\,\phi(2)\}|0\rangle\Big|_{\theta=\bar{\theta}=0} = T_\lambda\,\langle\phi(0)\rangle \qquad (3.6)$$

Thus, for every (nonhermitean) $\tilde{G}$ generator T_λ which is not annihilated by $\langle\phi\rangle$ the L.H.S. of (3.6) has to be non-zero. Now, because of superspace translation invariance one can write

$$\begin{aligned} &\langle 0|T\{J_\lambda(x_1,\theta_1,\bar{\theta}_1)\,\phi(x_2,\theta_2,\bar{\theta}_2)\}|0\rangle \\ &= \exp\{i(\theta_1\sigma^\mu\bar{\theta}_1+\theta_2\sigma^\mu\bar{\theta}_2-2\theta_2\sigma^\mu\bar{\theta}_1)\,\partial^{(2)}_\mu\} \\ &\quad\cdot\langle 0|\,J_\lambda(0,0,0)\;\phi(x_2-x_1,\theta_2-\theta_1,0)\,|0\rangle \end{aligned} \qquad (3.7)$$

assuming unbroken supersymmetry. Acting with $D^2\bar{D}^2$ on (3.7) and transforming to momentum space, one obtains for the $\theta_i = \bar{\theta}_i = 0$ component

$$\begin{aligned} &F.T.\; D^2\bar{D}^2\, e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}\,\langle 0|J_\lambda(0,0,0)\,\phi(-\theta)|0\rangle\Big|_{\theta=\bar{\theta}=0} \\ &= D^2(k)\,\bar{D}^2(k)\, e^{\theta\sigma^\mu\bar{\theta}k_\mu}\, f(\theta)\Big|_{\theta=\bar{\theta}=0} \\ &\sim k^2 \end{aligned} \qquad (3.8)$$

But because of (3.6) this does not vanish for $k^2 \to 0$. Hence we conclude that there is a $1/k^2$ pole in $\langle 0|J_\lambda(0)\phi(2-1)|0\rangle$, indicating a massless excitation coupling to the current J_λ. This proves the Goldstone theorem: for every broken $\tilde{G}$ generator T_λ there exists a massless excitation coupling to J_λ. Now, as we mentioned in the introduction, $T_\lambda\langle\phi\rangle \neq 0$ does not necessarily imply $T_{\lambda^*}\langle\phi\rangle \neq 0$, see e.g. (1.4). Therefore the Goldstone excitation does not, in general, couple to J_{λ^*}, i.e., J_λ and J_{λ^*} are really independent quantities. This leads to relate J_λ and J_{λ^*} separately to different Goldstone interpolating fields Π_λ and Π_{λ^*}. Before this is done in the next section, we remember that our starting point, (3.1), remains unchanged if we gauge an unbroken subgroup $S \subseteq H$. Thus, the Goldstone spectrum is not changed by this gauging, i.e., the radiative Goldstone mass shift is zero. This reflects just the nonrenormalization properties of massless fields [11], which are, however, not trivial if one deals with bound states. For other non-perturbative proofs of the absence of radiative mass shift for supersymmetric Goldstone fields see [12,17].

4. Current-Field Identities

In conventional theories there are the usual PCAC current-field identities which look close to mass shell

$$\langle 0 | J_{a\mu}(x) | \pi_b \rangle = -i k_\mu f_\pi \, \mathrm{Tr}(T_a T_b) \, e^{ikx} \tag{4.1}$$

$$\langle 0 | \partial^\mu J_{a\mu}(x) | \pi_b \rangle = -m_\pi^2 f_\pi \, \mathrm{Tr}(T_a T_b) \, e^{ikx} \tag{4.2}$$

(we normalize $\mathrm{Tr}(T_a T_b) = \delta_{ab}$). This implies the real Goldstone interpolating field operator to be

$$\hat{\pi}_a = \frac{-1}{m_\pi^2 f_\pi} \partial^\mu J_{a\mu} \tag{4.3}$$

(suppressing indices on m_π and f_π). We wish to generalize this relation. Since our pseudo-currents J_λ are "one-half" of the usual hermitean currents, we are led to identify near mass shell

$$\langle 0 | \tfrac{-i}{16} (D^2 \bar{D}^2 J_\lambda)(x,0,0) | \pi_{\lambda'} \rangle = -\tfrac{1}{2} m_\pi^2 f_\pi \, \mathrm{Tr}(T_\lambda T_{\lambda'}) \, e^{ikx} \tag{4.4}$$

suggesting that

$$\hat{\bar{\Pi}}_\lambda(x,\theta,\bar\theta) = \frac{i}{8 m_\pi^2 f_\pi} D^2 \bar{D}^2 J_\lambda(x,\theta,\bar\theta) \tag{4.5}$$

is the antichiral interpolating superfield operator. However, as noted in the introduction there are "non-fully doubled" cases where the Goldstone superfields sit in a complex representation of H. In such a case $m_\pi \equiv 0$, that is, there exists no real PCAC. Eq. (4.5) is not well defined in such a case. Of course, even in conventional nonsupersymmetric PCAC, m_π may be regarded as a regulator which appears in such a way in physical matrix elements so that the limit $m_\pi \to 0$ can be taken at the end of the calculation. Thus, one may expect that this happens also in the supersymmetric case. On the other hand, one can perform calculations using (4.1) instead of (4.2) without referring to m_π explicitly. Thus it would be useful to have a supersymmetric generalization of (4.1). This will be discussed further below.

The problem of $m_\pi \equiv 0$ can be by-passed also in another way. As it will turn out below, inspite of the form of (4.5) only the true Goldstone boson component operators are proportional to $1/m_\pi^2$. For the fermionic components the $1/m_\pi^2$ dependence cancels out. Now, for the bosonic components of Π, $m_\pi \neq 0$ is not forbidden by any chiral symmetry. Only the quasi-Goldstone fermions may be chiral protected. Thus, we can regard $m_\pi \neq 0$ as an (explicit supersymmetry violating) bosonic regulator mass which is set to zero at the end of a calculation.

(4.5) can also be derived by acting with the anti-chiral projector $1/16\,\Box^{-1}D^2\bar{D}^2$ on the m_π independent identity

$$\langle 0 |\, J_\lambda(x,\theta,\bar\theta)\, | \pi_{\lambda'} \rangle = -i\, f_\pi\, \mathrm{Tr}(T_\lambda T_{\lambda'})\, e^{-i\theta\sigma^\mu\bar\theta\partial_\mu}\, e^{ikx} \tag{4.6}$$

This is a more fundamental relation, from which generalizations of (4.1) and (4.2) can be derived by acting on it with covariant derivatives. One can immediately identify the component interpolating field operators. For simplicity, we focus on the case of canonical kinetic terms in the following. With the definition of the components of the current superfield

$$\begin{aligned} J_\lambda(\theta,\bar\theta) = {} & C_\lambda + \theta^\alpha J_{\lambda\alpha} + \bar\theta^{\dot\alpha} \bar{J}_{\lambda\dot\alpha} + \tfrac{1}{2}\theta\theta M_\lambda + \tfrac{1}{2}\bar\theta\bar\theta M^*_\lambda \\ & + \theta^\alpha \sigma^\mu_{\alpha\dot\alpha} \bar\theta^{\dot\alpha} J_{\lambda\mu} + \tfrac{1}{2}\bar\theta\bar\theta\theta^\alpha \Lambda_{\lambda\alpha} + \tfrac{1}{2}\theta\theta\bar\theta^{\dot\alpha}\bar\Lambda_{\lambda\dot\alpha} \\ & + \tfrac{1}{4}\theta\theta\bar\theta\bar\theta D_\lambda \end{aligned} \tag{4.7}$$

one obtains

$$\langle 0 | \; C_\lambda(x) \, | \pi_{\lambda'} \rangle = -i \, f_\pi \, \mathrm{Tr}\,(T_\lambda T_{\lambda'}) \, e^{ikx} \tag{4.8}$$

$$\langle 0 | \; \bar{J}_{\lambda\dot{\alpha}}(x) \, | \psi_{\lambda'\alpha} \rangle = 2 \, k_{\alpha\dot{\alpha}} \, f_\pi \, \mathrm{Tr}(T_\lambda T_{\lambda'}) \, e^{ikx} \tag{4.9}$$

Thus, the anti-quasi-Goldstone fermion operator is given by the spinor current

$$\hat{\psi}^{\dot{\alpha}}_{\lambda} = \frac{1}{f_\pi} \, \bar{J}^{\dot{\alpha}}_{\lambda} \tag{4.10}$$

In fact, there is no dependence on m_π , so that this relation is well defined even for chiral protected fermions. On the other hand, the bosonic operator is not given by just C_λ . (4.8) involves only the lowest component of J_λ and does not include derivatives. Acting with $D\bar{D}$ on (4.6) gives the momentum dependent identity, generalizing (4.1):

$$\langle 0 | \, J_{\lambda\mu}(x) + k_\mu \, C_\lambda(x) \, | \pi_{\lambda'} \rangle = -2i \, k_\mu \, f_\pi \, \mathrm{Tr}\,(T_\lambda T_{\lambda'}) \, e^{ikx} \tag{4.11}$$

The second term of the L.H.S. contains just the momentum-independent identity (4.8). Thus, one obtains for the anti-Goldstone boson operator near mass shell

$$\hat{\pi}^{*}_{\lambda} = \frac{-1}{m_\pi^2 f_\pi} \, \partial^\mu J_{\lambda\mu} + \frac{i}{f_\pi} \, C_\lambda \tag{4.12}$$

and for its real and imaginary parts

$$m_\pi^2 f_\pi \, \mathcal{R}e\,(\hat{\pi}^{*}_{\lambda}) = - \partial^\mu J^{+}_{\mu} \left(\frac{\lambda+\lambda^*}{2}\right) + i \, m_\pi^2 \, C^{-} \left(\frac{\lambda-\lambda^*}{2}\right) \tag{4.13}$$

$$m_\pi^2 f_\pi \, \mathcal{I}m\,(\hat{\pi}^{*}_{\lambda}) = - \partial^\mu J^{-}_{\mu} \left(\frac{\lambda-\lambda^*}{2}\right) + i \, m_\pi^2 \, C^{+} \left(\frac{\lambda+\lambda^*}{2}\right) \tag{4.14}$$

The R.H.S. of (4.13) and (4.14) are precisely the $\theta = \bar{\theta} = 0$ components of (2.19) and its conjugate, respectively. One can interpret (4.13) and (4.14) as follows: The $\partial^\mu J_\mu$ parts represent just true Goldstone operators [F4] ($\hat{g}$), as can be seen from the derivative low energy decoupling property.

The C parts are pseudo-Goldstone operators ($\hat{p}$). Thus, in general, both real and imaginary parts of π have both pseudo- and true Goldstone boson properties.

Now, we come to an important point. Namely, the question which basis $\{T_\lambda\}$ is to be chosen for a specific problem. One has to distinguish two cases. First, as reviewed in the introduction, there exists the "full doubling" case with equal numbers of pseudo- and true Goldstone bosons. This situation precisely obtains if for any broken $\tilde{G}$-generator

$$T_\lambda \langle\phi\rangle \neq 0 \tag{4.15}$$

also its hermitean conjugate is broken

$$T_{\lambda^*} \langle\phi\rangle \neq 0 \tag{4.16}$$

This means the theory is symmetric in J_λ and J_{λ^*} [F5]. Hence, we can choose a hermitean basis for the broken generators, i.e., $J_\lambda = J_{\lambda^*} = J^+_\lambda = J^{conv.}_\lambda$, $J^-_\lambda = 0$ for canonical kinetic terms. Then we regain the conventional case which has been studied already in [8,9]. From (4.13) and (4.14) it follows

$$\mathcal{R}e\,(\hat{\pi}_\lambda) = \hat{g}_\lambda = \frac{-1}{f_\pi m_\pi^2}\,\partial^\mu J^{conv.}_{\lambda\mu} \tag{4.17}$$

$$Im\,(\hat{\pi}_\lambda) = i\hat{p}_\lambda = -\frac{i}{f_\pi}\, C^{conv.}_\lambda \tag{4.18}$$

Thus, in the "fully doubled" case the real part of π is the conventional Goldstone field (g), which couples only derivatively. On the other hand, the imaginary part does not couple derivatively, so that no low energy decoupling theorem is associated with it. We can identify it with a pseudo-Goldstone field (p), associated with the broken generators of $\tilde{G}/G$. (4.18) can also be derived directly from (4.8) which implies because of the behavior under complex conjugation, i.e., parity

$$\langle 0|\, C_\lambda(0)\, |p_\lambda\rangle = -f_\pi\,, \quad \langle 0|\, C_\lambda(0)\, |g_\lambda\rangle = 0 \tag{4.19}$$

and combined with (4.11) [F6]

$$\langle 0 | \partial^\mu J_{\lambda\mu}(0) | g_\lambda \rangle = -m_\pi^2 f_\pi \ , \ \langle 0 | \partial^\mu J_{\lambda\mu}(0) | \rho_\lambda \rangle = 0 \qquad (4.20)$$

(4.17) and (4.18) reflect precisely the well-known fact [1] that the low energy interactions in the sector of true Goldstone bosons are uniquely fixed in terms of G/H. The interactions of the pseudo-Goldstone bosons are not determined by the geometry of G/H and depend specifically on the linear theory.

Now we turn to the "non-fully doubled" case, for example (1.4). Here the number of Goldstone superfields is less than dim(G/H). This is due to the fact that

$$T_\lambda \ \langle \phi \rangle \neq 0 \qquad (4.21)$$

but nevertheless

$$T_{\lambda^*} \langle \phi \rangle = 0 \qquad (4.22)$$

From the Goldstone theorem it follows that J_λ is a "broken" current while J_{λ^*} is not. Thus, we have only a Goldstone superfield Π_λ but no Π_{λ^*}. Because of this asymmetry, we cannot choose a hermitean basis for the broken $\tilde{G}$-generators, i.e., J_λ is a pseudo-current. From (4.13) and (4.14) it follows that the real and imaginary parts of π do not obey (4.17) and (4.18) any more. At first sight it seems that because of (4.13) and (4.14) both real and imaginary parts contain both contributions from true ($\partial^\mu J_\mu$) and pseudo- (C) Goldstone bosons. However, since only J_λ is broken and J_{λ^*} not, we have $f_{\pi_\lambda} \neq 0$, $f_{\pi_{\lambda^*}} = 0$. From (4.13)

$$m_\pi^2 f_{\pi_\lambda} \mathcal{R}e(\hat{\pi}_\lambda) = -\partial^\mu J^+_\mu\left(\frac{\lambda+\lambda^*}{2}\right) + i m_\pi^2 C^-\left(\frac{\lambda-\lambda^*}{2}\right) \qquad (4.23)$$

follows

$$0 = m_\pi^2 f_{\pi_{\lambda^*}} \mathcal{R}e(\hat{\pi}_{\lambda^*}) = -\partial^\mu J^+_\mu\left(\frac{\lambda+\lambda^*}{2}\right) - i m_\pi^2 C^-\left(\frac{\lambda-\lambda^*}{2}\right) \qquad (4.24)$$

which fixes a relation between $\partial^\mu J^+_\mu(\lambda+\lambda^*)$ and $C^-(\lambda-\lambda^*)$. Similar arguments apply also to $\mathrm{Im}(\hat{\pi}_{\lambda^*})$. Thus, we can eliminate $C^-(\lambda-\lambda^*)$ in

(4.23) and obtain for the real and imaginary parts of π_λ

$$\begin{aligned} \mathrm{Re}\,(\hat{\pi}_\lambda) &= \frac{-1}{m_\pi^2 f_\pi}\, \partial^\mu J_\mu^+ (\lambda + \lambda^*) = 2\, \hat{g}_{\lambda+\lambda^*} \\ \mathrm{Im}\,(\hat{\pi}_\lambda) &= \frac{1}{m_\pi^2 f_\pi}\, \partial^\mu J_\mu^- (\lambda - \lambda^*) = 2i\, \hat{g}_{\lambda-\lambda^*} \end{aligned} \tag{4.25}$$

Both behave as true Goldstone bosons, as it was expected by simple counting arguments indicated in the introduction. Moreover, what happens if one gauges an unbroken subgroup S $\subseteq$ H? It is clear that some of the Goldstone bosons turn into pseudo-Goldstone bosons, since some symmetries transform into pseudo-symmetries. This was discussed in Section 2, where it was shown that the only change is that the current J_λ is now of the form (2.31). Hence, the PCAC-relations (4.4)-(4.6) remain unchanged. The pseudo-nature of the Goldstone bosons appears only at the component level. For instance, it follows from (2.29) that in the fully doubled case the true Goldstone operator (4.17) is modified to a pseudo-Goldstone operator:

$$f_\pi m_\pi^2\, \hat{g}_\lambda = -\partial^\mu J_\mu^+ + \frac{i}{32}\left(\{D_1^2, \bar{D}^2\}\, \bar{\phi}\,[e^V, T_\lambda]\,\phi \right)\Big|_{\theta=\bar{\theta}=0} \tag{4.26}$$

To summarize, we derived SUSY generalizations of the conventional PCAC current field identities. These generalizations imply the usual true Goldstone low energy theorems and weaker theorems for pseudo-Goldstone bosons.

5. Dashens's Formula

We now want to derive a SUSY variant of Dashen's formula [14]. For fully doubled cases, this has been done already in Refs. [9,10,15,17]. Our derivation, however, applies also to the general case. In some parts, we follow the strategy of [10]. To begin, consider the Green's function

$$\int dx_1 \langle 0 | \tau \{ D^2(1)\, \bar{D}^2(1)\, J_\lambda(1)\, \delta_{\chi} W(2) \} | 0 \rangle \tag{5.1}$$

where $\delta_\chi W(2) \neq 0$ expresses the explicit breaking of the global symmetry $\tilde{G}$. Using the Ward identity (2.12a) and the action principle

$$\langle 0|\tau\{\delta\phi \frac{\delta\Gamma}{\delta\phi}X\}|0\rangle = i\langle 0|\tau\{\delta\phi \frac{\delta X}{\delta\phi}\}|0\rangle \tag{5.2}$$

and noting that (see (2.10))

$$\begin{aligned} &-\frac{1}{4}\int dx_1 \langle 0|\tau\{D^2(1)\delta_\lambda \phi(1)\frac{\delta}{\delta\phi(1)}(\delta_{\lambda'} W(2))\}|0\rangle \\ &= \langle 0|\hat{\Omega}_\lambda(\phi)\,\delta_{\lambda'}W(2)|0\rangle \\ &= \langle 0|\delta_\lambda\delta_{\lambda'}W(0)|0\rangle \end{aligned} \tag{5.3}$$

one arrives at

$$\begin{aligned} &\frac{i}{16}\int dx_1 \langle 0|\tau\{D^2(1)\bar{D}^2(1)J_\lambda(1)\,\delta_{\lambda'}W(2)\}|0\rangle \\ &= \frac{1}{4}\int dx_1 \langle 0|\tau\{D^2(1)\,\delta_\lambda W(1)\,\delta_{\lambda'}W(2)\}|0\rangle \\ &+ i\langle 0|\delta_\lambda\delta_{\lambda'}W(0)|0\rangle \end{aligned} \tag{5.4}$$

Now, we transform to momentum space and insert Goldstone intermediate states into the first two terms, saturating the spectrum. Comparing the residues on shell yields

$$\begin{aligned} &F.T.\left(\frac{i}{16}\langle 0|D^2\bar{D}^2J_\lambda|\pi\rangle\langle\pi|\delta_{\lambda'}W|0\rangle\right) \\ &= F.T.\left(\frac{1}{4}\langle 0|D^2\delta_\lambda W|\pi\rangle\langle\pi|\delta_{\lambda'}W|0\rangle\right) \end{aligned} \tag{5.5}$$

Inserting the current field identity (4.4) we have, therefore,

$$\langle 0|D^2\delta_\lambda W(0)|\pi\rangle = 2m_\pi^2 f_\pi \tag{5.6}$$

Taking now the zero momentum limit of (5.4), the first term vanishes assuming no massless pole because of the perturbation $\delta_\chi W \neq 0$. Hence

$$\frac{1}{4}\langle 0|D^2\delta_\lambda W|\pi\rangle \frac{1}{m_\pi^2}\langle\pi|\delta_{\lambda'}W|0\rangle = -i\langle 0|\delta_\lambda\delta_{\lambda'}W|0\rangle \tag{5.7}$$

In [10] it was shown that

$$|\langle 0|\delta_\lambda W|\pi\rangle|^2 = \frac{1}{m_\pi^2}\,|\langle 0|D^2\delta_\lambda W|\pi\rangle|^2 \tag{5.8}$$

Combining (5.6)-(5.8) yields finally, after introducing indices on f_π and m_π

$$(m_\pi)_{\lambda\lambda'} = \frac{-i}{f_{\pi_\lambda} f_{\pi_{\lambda'}}}\langle 0|\delta_\lambda \delta_{\lambda'} W(0)|0\rangle \tag{5.9}$$

as generalization of Dashen's formula. It has the usual form [9,10,15,17]. But its derivation was more general so that it is valid also for these "non-fully doubled" cases where Goldstone superfields are real under H. Of course, in cases where Goldstone superfields are complex with respect to H, $m_\pi \equiv 0$, that is, both sides of (5.9) vanish identically. As indicated in Section 4, in (5.7) and (5.8) m_π should be regarded as a bosonic regulator mass which is finally put to zero, so that (5.7) and (5.8) are well defined. We want to note that although one could replace $\delta_\lambda W$ by $\delta_\lambda L$ in (5.1), one could not use (5.8) to eliminate $\langle 0|\delta L|\pi\rangle$ in (5.7). Thus, according to this derivation, Dashen's formula involves only the superpotential and not the whole Lagrangian. Of course, this is not a proof but it is plausible since as stated above, the Goldstone spectrum depends only on properties of the superpotential.

6. Summary and Further Aspects

In the foregoing sections, we emphasized the relevance of global superpotential symmetries in supersymmetric Goldstone theories. We introduced pseudo-symmetry currents obeying generalized conservation laws. These pseudo-currents are basically "one-half" of the conventional hermitean currents; this feature is essential if one wants to describe Goldstone fields which are asymmetric, i.e., non-real with respect to the unbroken subgroup. More precisely, for every symmetry of the superpotential there exists such a pseudo conservation law. Of course, among these superpotential symmetries are the "true" symmetries under which the whole Lagrangian including D-terms is invariant. For these symmetries, the pseudo-conservation laws turn into conventional current conservation laws, associated with conserved charges. However, we showed that in fact just the larger class of pseudo-symmetry currents is relevant for Goldstone physics. This was proven by deriving Goldstone's theorem using the more general Ward identities for pseudo-symmetry currents, which involve only the superpotential. Specifically, gauging an unbroken subgroup does not change the superpotential symmetry. Thus, the pseudo-current conservation laws are not affected. Then, by Goldstone's theorem, this immediately leads to the conclusion that Goldstones get no radiative mass shift by gauge fields. Moreover, we presented a general framework of supersymmetric PCAC. Our formulation also applies to cases where the number of chiral Goldstone superfields N_π differs from dim(G/H). The structure of Goldstone field operators comes out as naively expected: In the "fully doubled" case where N_π = dim G/H, the real part of the scalar component is just the usual Goldstone boson, subject to low energy decoupling theorems. The imaginary part is a pseudo-Goldstone boson associated with the breaking of pseudo-symmetries of the superpotential. Its interactions are not restricted by strong low energy theorems. In cases where $N_\pi <$ dim(G/H), we showed that both real and imaginary parts decouple at low momenta, which is consistent with their interpretation as to be both true Goldstone bosons. Finally, we derived a further generalization of Dashen's formula, valid for all values of N_π .

Of course, there are many further aspects which were not discussed yet and some of which are presently under investigation. For instance, in [8] it was pointed out that if the conventional hermitean current $J_\lambda^{\text{conv.}}$ had a vacuum expectation value,

$$\langle 0 | J_\lambda^{(\text{conv})}(0) | 0 \rangle \neq 0 \tag{6.1}$$

peculiar things could happen. Now, looking to the matrix element of conventional currents

$$F.T. \quad \langle 0 | \tau \{ \partial^\mu J_{\lambda'\mu}^{\text{conv.}} \, J_\lambda^{\text{conv.}} \} | 0 \rangle \tag{6.2}$$

and using (2.22) and the action principle (5.2) one immediately gets some sort of Goldstone's theorem:

$$k^\mu \langle 0 | \tau \{ J_{\lambda'\mu}^{\text{conv.}} \, J_\lambda^{\text{conv.}} \} | 0 \rangle \Big|_{\theta = \bar\theta = 0} = \delta_{\lambda'} \langle 0 | J_\lambda^{\text{conv}} | 0 \rangle \tag{6.3}$$

Thus, for any symmetry broken by (6.1) there has to be a massless excitation coupling to $J_\lambda^{conv.}$ and $\partial^\mu J_{\lambda\mu}^{conv.}$ which however are both real. Hence, we argue that this excitation belongs also to a real supermultiplet $\Pi = \bar{\Pi}$, i.e., to a new kind of supersymmetric Goldstone excitation. Its structure should be equal to that of the symmetry breaking field $J_\lambda^{conv.}$.

Another point is the question of anomalies: are there anomalous contributions to the pseudo-current Ward identities (2.12) at the quantum level? In the case of supersymmetric QCD it is well known that the anomalous contributions split in fact into two parts [16],

$$\begin{aligned} \delta_{\Lambda,\bar{\Lambda}} \Gamma &= \hat{\Omega}_\Lambda(\phi)\Gamma + \hat{\Omega}_{\bar{\Lambda}}(\bar{\phi})\Gamma \\ &= G(\Lambda) + \bar{G}(\bar{\Lambda}) \end{aligned} \tag{6.4}$$

where Λ is the chiral superparameter of axial gauge transformations and $G(\Lambda)$ the generalization of the chiral Adler-Bardeen anomaly given by [16]

$$G(\Lambda) = \frac{iN_c}{32\pi^2}\int dS\, Tr(\Lambda W^\alpha W_\alpha) \tag{6.5}$$

(Here N_C is the number of colors and W_α is the $SU(N)_V$ chiral field strength spinor.) Thus, because of the separation of Λ and $\bar{\Lambda}$ dependent parts in (6.4) we argue that our pseudo-current conservation law (2.12a) gets anomalous contributions of the form

$$D^2\bar{D}^2 J_\lambda = \frac{N_c}{8\pi^2} D^2 Tr(T_\lambda W^\alpha W_\alpha) \tag{6.6}$$

The implications are not clear yet. Perhaps, one has to extend the concept of anomaly matching. We hope to come back to these issues in a further publication.

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Footnotes

[F1] This case cannot be realized by a model of type (1.1). However, after introduction of gauge fields it is possible in some cases [6,4]. Of course, a necessary condition is that G/H is Kählerian.

[F2] This generalized divergence is a sum of "counting operators" [10] which vanish upon using the equations of motion. This may lead to simplifications in calculating Green's functions, as was emphasized in [10].

[F3] In fact, after introducing S gauge fields there are additional terms in the Ward identities (2.12), proportional to δV. However, in the context of Goldstone physics we are interested only in the broken global $\tilde{G}/\bar{S}$ pseudo-symmetries since broken local symmetries $\bar{S}$ do not lead to Goldstone fields because of the Higgs mechanism. But for these global $\tilde{G}/\bar{S}$ symmetries $\delta V = 0$. Thus Eqs. (2.12) remain unchanged for $\tilde{G}/\bar{S}$ symmetry currents. Note we write $\bar{S}$ instead of S since the full local symmetry is the complex extension of S.

[F4] Strictly speaking, if $m_\pi \neq 0$ all Goldstones are pseudo. However, we use the terms true and pseudo- Goldstone bosons according to their origin in the massless limit, i.e., true or pseudo- symmetries.

[F5] However, in the general case the VEV's in (4.15) and (4.16) need not correspond to the same field. Thus, T_λ and T_{λ^*} may be broken at different scales, implying $f_{\pi_\lambda} \neq f_{\pi_{\lambda^*}}$. Then the theory is not symmetric in J_λ and J_{λ^*}. We assume here that $f_{\pi_\lambda} = f_{\pi_{\lambda^*}}$ which is implied automatically if e.g. $\langle\phi\rangle$ is in a real irreducible representation of G or G/H is a symmetric space [13].

[F6] For $m_\pi = 0$, both equations in (4.20) look equivalent. However, the last relation should be interpreted that there is no one-particle state $|p_\lambda\rangle$ coupling to $\partial^\mu J_{\lambda\mu}$, contrary to $|g_\lambda\rangle$.

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