# Recent Developments in String Theory

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**Overview in german:**

Die Stringtheorie hat in den letzten Jahren einen großen Aufschwung genommen. Sie ist ein Gebiet der theoretischen Physik, welches sich die Vereinheitlichung aller Naturkräfte zum Ziel genommen hat. Diese sind bekanntlich durch den Elektomagnetismus, die starke und die schwache Wechselwirkung sowie durch die Gravitation gegeben.

In der Stringtheorie wird angenommen, daß die tiefste zugrundeliegende Struktur aller Elementarteilchen und ihrer Wechselwirkungen durch ultrakleine "strings" oder Fädchen gegeben ist. Die verschiedenen in der experimentellen Beschleunigerphysik beobachtbaren Teilchen werden als Anregungen (besser Nullmoden) dieser Strings interpretiert. Diese Vorstellung ist keineswegs ad hoc vom Himmel gefallen, sondern gründet sich auf die Einsichten und Schwierigkeiten, auf die man mit Versuchen der Quantisierung der Gravitation gestoßen ist. Man fand, daß sich in der Stringtheorie alle Inkonsistenzen wie durch ein Wunder in Nichts auflösen, und entdeckt immer weitere frappierende Eigenschaften, welche die Theorie auszeichnen. Daher wird allgemein geglaubt, daß die Probleme der Quantengravitation nur in der Stringtheorie umgangen werden, und als "Nebenprodukt" findet man automatisch eine vollkommene Vereinheitlichung der restlichen drei Naturkräfte.

Allerdings sind diese Überlegungen bisher rein theoretischer, und vor allem eher mathematischer Natur. Daß dies wohl auch in der näheren Zukunft so bleiben wird, liegt an den riesigen Energieskalen, die involviert sind – ein "praktisches" Problem, mit welchem jede Theorie einer Vereinheitlichung der Naturkräfte zu kämpfen hat. Daher sucht man vor allem nach theoretischen Konsistenz- und Eindeutigkeitsbeweisen. In der letzten Zeit wurden hier große Fortschritte gemacht, die auch auf konventionelle Quantenfeldtheorien, wie Eichtheorien, ausstrahlen. Das Stichwort heißt "Dualität", welches eine exakte, nichtperturbative Äquivalenz a priori verschiedener Quantentheorien bedeuted. Erheblichen spinoff gibt es auch in die Richtung der reinen Mathematik (zB. algebraischen Geometrie), so daß man fast sagen könnte: die Stringtheorie ist so reichthaltig, daß sie im Grunde nicht nur alle bekannte Physik umfaßt, sondern auch einen großen Teil der bekannten Mathematik.

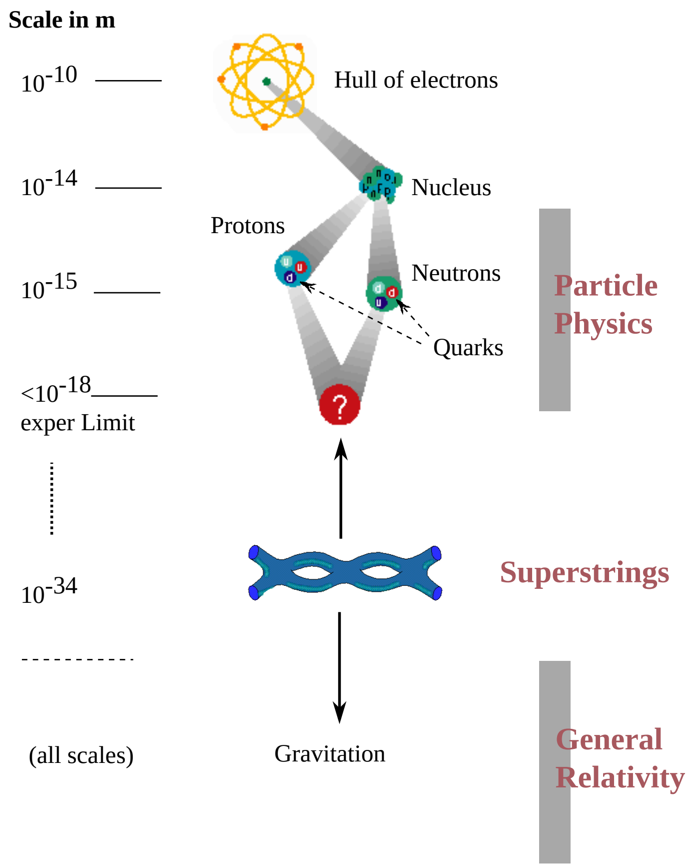



Figure 1: Superstrings form the bridge between particle physics and gravity. Our knowledge from experiments ends somewhere at $10^{-18}m$, and from there it's still a long way to go until we could possibly meet superstrings.

# 1 Introduction

## 1.1 The Standard Model of Particle Physics, and its Deficiencies

String theory[1] is an approach to describe the constituents of all matter and their interactions, at the most fundamental and deepest level. It is thus an extension of elementary particle physics, and in fact most likely the only possible extension that is consistent with both quantum mechanics and general relativity. To get a quick idea about the relevant distance scales we will talk about, see Fig.1.

Conventional particle physics deals with the properties of the building blocks of matter, namely quarks and leptons –of which the most prominent example is the electron–, and their (non-gravitational) interactions. There are three such interaction forces, namely the well-known electromagnetic force, plus the "weak" and the "strong" interactions. The latter two play a rôle only in subatomic physics, and manifest themselves macroscopically in an indirect manner, like via radioactive decay. According to quantum field theory, all such forces are mediated by certain force carrier fields, nowadays called gauge fields. The most prominent gauge field is the photon, the quantum of light, which gives rise to the electromagnetic interaction. Analogously, for the other interactions there are "$W$-bosons" and "gluons", respectively.

[1] For a recent account intended for general audience, see [1], and for more technical reviews, see for example [2].

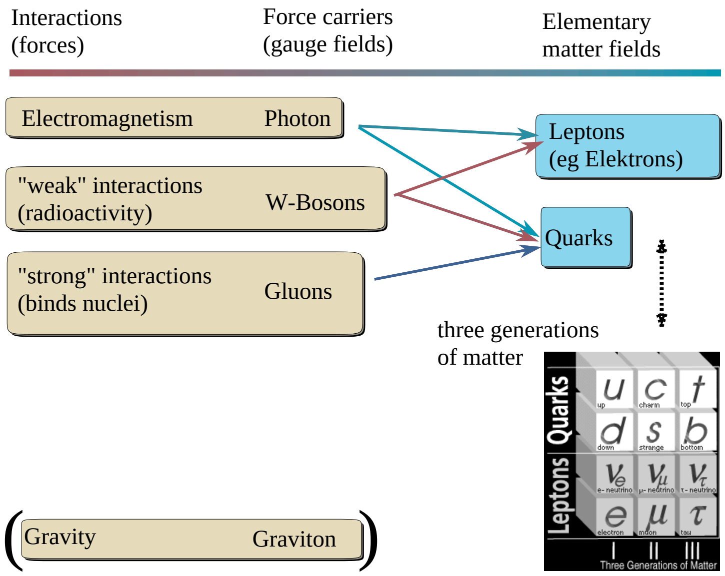



Figure 2: The standard model of particle physics comprises the three observed families of quarks and leptons, together with certain interactions that are mediated by gauge fields. ($\nu$ denotes the various kinds of neutrinos, and $e$ the electron). The fourth known interaction force of nature, which is gravity, is traditionally not viewed as belonging to particle physics.

The observed elementary particles and their interactions are described by what is called the "Standard Model" of particle physics. This is a certain framework of formulas and field theoretic rules, developed over the past 50 years, by which we can describe many subatomic phenomena with stunning accuracy. We have depicted the most important ingredients of the Standard Model in Fig.2.

However, even though this theoretical framework is very successful (in fact it is the most precise and accurate theory in all of natural sciences), there is quite a number of partly technically, and partly conceptually unsatisfying deficiencies.

One of the first questions that comes to mind is the very structure of the standard model, which seems to be quite ad hoc. For example, one may ask why do we have just three, and not four, of quark-lepton families, as depicted in Fig.2 ? Why is the structure of their gauge interactions just as we see it, and not something else ? Moreover, there are circa 20 free parameters in the Standard Model, like the electron mass, whose observed numerical values are unexplained per se, and so must by fitted against the experimental data. So, what is the reason –if there is any at all– for these parameters to take exactly the numerical values we observe ?

What physicists are hoping for is that there should be some structure underlying the Standard Model, which would explain some or all of these and other features. Indeed, numerous extensions of the Standard Model have been contemplated upon in the past, like the "Grand Unification" of the gauge interactions, or "supersymmetry". Supersymmetry by itself does actually not solve the above-mentioned conceptual problems, but provides an attractive escape route to avoid certain technical problems (among other features, it softens self-energy corrections to particle masses, thereby providing a mechanism for the stability of small mass values). It is a symmetry between bosonic (integer spin, like that of photons) and fermionic degrees (half-integer spin, like that of electrons) of freedom - see Fig.3.

Unfortunately the energy range and accuracy of the present-day accelerator experi-

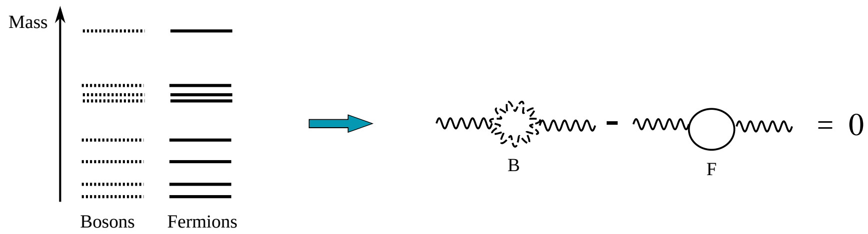



Figure 3: In supersymmetric field theories, the fermionic spectrum matches the bosonic one. Since fermionic and bosonic contributions to self-energies have opposite signs, they tend to cancel out to the effect that the theory less is divergent as compared to a non-supersymmetric theory.

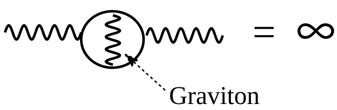



Figure 4: Graviton scattering processes, like this self-interaction, cannot be made sense of in conventional quantum field theory because of incurable divergences.

ments did not yet allow us to "see" any territory beyond the Standard Model (and this despite of the fact that the involved machinery is among the largest and most sophisticated that ever has been made). However, it is guaranteed that the next generation of particle accelerators and detectors, like the LHC currently being built at CERN, will shed light on some of this uncharted area, most notably on the question of supersymmetry (near the question mark in Fig.1).

### 1.2 Quantum gravity and String Theory

The most dramatic extension of the Standard Model of particle physics that has been proposed so far is string theory. However, as we will discuss in more detail below, string theory too does not provide very concrete answers to the questions posed above. But what string theory does is to provide a resolution of conceptual problems that are on a far deeper level than these "practical" problems. One of the most important problems in modern theoretical physics is the apparent mutual incompatibility of quantum mechanics and general relativity (the theory of gravity) – one theory describing well the world at very short, the other at long distances. Certainly a truly satisfying unified theory should incorporate the gravitational interaction as well, even though traditionally it is not considered as belonging to particle physics.

Over the last decades, countless attempts had failed in trying to formulate a theory of gravity at the quantum level. For example, if we try to apply the quantum field theoretical methods that were so successful in particle physics, we get answers that make no sense, even for simple interaction processes. More specifically, from the point of view of quantum field theory, the gravitational force is mediated by gravitons which are the quanta of the gravitational field. However, scattering processes involving such gravitons, like the one depicted in Fig.4, always lead to infinite answers (due to "non-renormalizable" divergences).

This failure is just a reflection of a more deeply rooted problem, namely an actual inconsistency between standard quantum field theory and gravity, which shows up in many disguises. It has been clear for some while, therefore, that in order to bypass

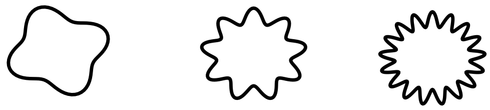


Figure 5: Some vibrational excitation modes of a tiny closed string.

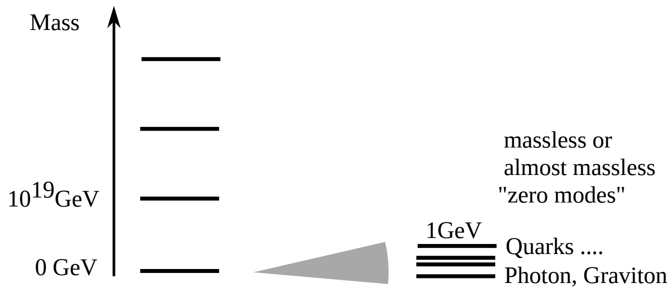



Figure 6: The energy level spacing of the string vibration modes is very large, so that what we usually call "high energy physics" is really at very low energies from the string point of view. The characteristic scale of the string excitations, given by the "Planck mass" $m_{\text{Planck}} \sim 10^{19}$ GeV, is set by the strength (rather: weakness) of the gravitational interaction at low energies.

these obstacles, entirely new concepts at a most fundamental level are needed. It is in string theory, where indeed –as far as we can see– all these inconsistencies miraculously disappear, and it is widely believed that it is in fact the only possible framework in which a marriage between quantum mechanics and general relativity can be achieved.

Just this fact by itself could be taken as a revolutionary breakthrough in theoretical physics, but in addition string theory brings with it –in some sense as a by-product– the generic features of the Standard Model as well. More precisely, it carries generically also the types of degrees of freedom that we actually see in Nature, like quarks, gauge and Higgs fields etc. However, apart from this generic structure, the details, like particle masses, precise form of gauge groups etc, cannot (at least as of yet) predicted in any concrete way. As will be explained below, these details do not seem to be encoded in the string theory at a fundamental level, but are governed by the specific vacuum ground state of the theory. At the moment it thus seems very hard to go beyond this point and derive the numerical details of the standard model from first principles. Perhaps it will never be possible, because there might simply be no such first principles, the numerical details being nothing but frozen historical accidents.

Heuristically one may introduce string theory by imagining tiny, oscillating loops sweeping through space-time. The vibrational excitation modes, of which there are infinitely many, then would correspond to "elementary particles". The characteristic length must however be extremely small; correspondingly, the level spacing of the excitation spectrum must be huge, and in fact of the order of $10^{19}$ proton masses – see Figs.5 and 6. This is simply because this mass scale is ultimately determined by the value of the gravitational coupling constant, and this must match because we do want to describe gravity correctly.

That strings are so difficult to observe and verify experimentally, thus stems from the fact that the gravitational coupling is so small. Were we able to access this mass scale, there would be infinitely many testable predictions that were specific to string theory; it is not at the fault of string theory that we cannot access them with today's technology.

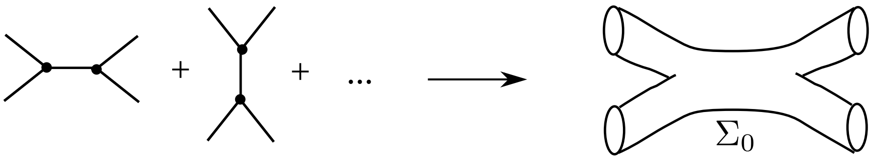
In fact, the experimental verification *any* grand unified theory, especially if it includes gravity, tends to suffer from this fact, so that this problem is not string-specific. Still, it is not so that string theory would not make any predictions at low-energies; as we will see, the theories are very finely tuned and internal consistency still dramatically reduces the number of possible low energy spectra and independent couplings, as compared to ordinary field theory.

Moreover, if we are lucky we might find out that the situation is not that bad; indeed, certain string inspired models have been conceived recently [3], in which the string scale (and the scale of extra dimensions) lies within the range of the next-generation accelerator experiments (or even at the millimeter scale). However, there seems to be no particularly convincing reason why Nature should have chosen a string model that by accident happens to be just about verifiable by present-day experiments.

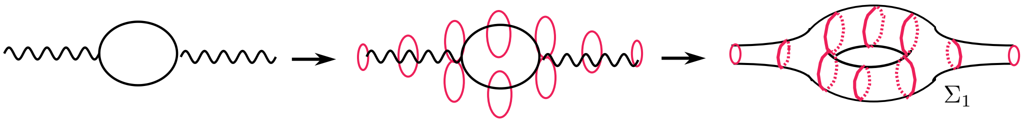
# 2 Perturbative String Theory

## 2.1 Two-Dimensional World-Sheets

Being one-dimensional extended objects immersed in some $D$-dimensional[2] space-time, strings trace out two-dimensional "world-sheets" $\Sigma$ that may be viewed as thickened Feynman diagrams, for example:




Analogously, a self-interaction process is described by a two-dimensional surface with a hole:




For a long time, the only known way to think about strings was in terms of such two dimensional Riemann surfaces, somewhat neglecting that this represents only an approximation scheme, whose validity requires surfaces with more and more holes to give less and less strong contributions to a given scattering process. In the recent "second string revolution" this perturbative scheme based on Riemann surfaces has been overcome, which now allows highly non-trivial and important insights into the exact, non-perturbative properties of string theories – this will be the topic of subsequent sections.

In the perturbative formulation that we consider momentarily, a definition of string theory can relatively simply be given in terms of two-dimensional field theories living on the Riemann surfaces. Suitable two-dimensional field theories can be made out of a great variety of building blocks, the simplest possibilities being given by free two-dimensional bosons and fermions, eg., $X_i(z)$, $\psi_i(z)$.

By combining the two-dimensional building blocks according to certain rules, an infinite variety of operators that describe *space-time* fields can be constructed. These correspond to the zero modes and massive excitations of the string, cf. Fig.6). Typically, the more complex the operators become that one builds, the more massive the associated space-time fields become. In total one obtains a finite number of massless fields, plus an infinite

[2] We prefer to leave the space-time dimension at this point arbitrary, aiming at $D = 4$ at a later time.

tower of string excitations with arbitrary high masses and spins. Schematically, among the most generic massless operators are:

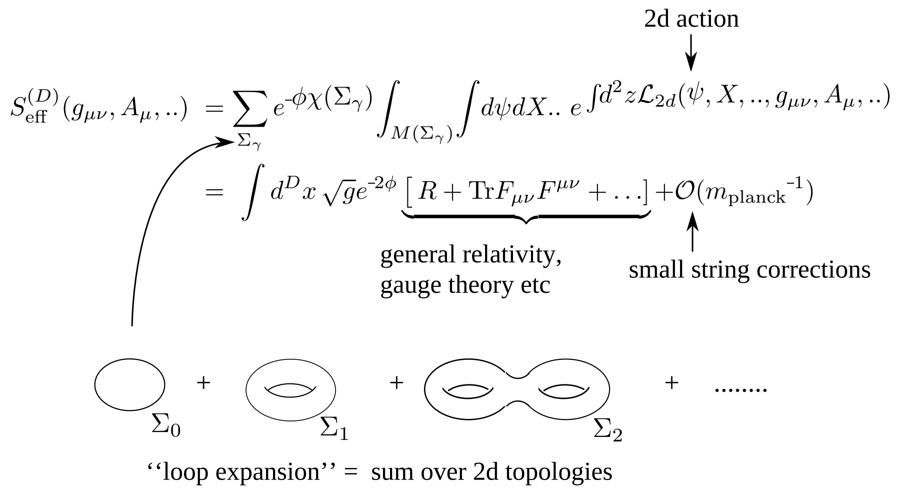
$$\begin{array}{rll} \text{Dilaton scalar} & \phi & \eta^{\mu\nu}\bar{\partial}X_\mu\partial X_\nu \qquad (\mu,\nu=1,...,D) \\ \text{Graviton} & g_{\mu\nu} & \bar{\partial}X_\mu\partial X_\nu \\ \text{Gauge field} & A^a_\mu & \bar{\partial}X_\mu\partial X^a \\ \text{Higgs field} & \Phi^{ab} & \bar{\partial}X^a\partial X^b \end{array} \tag{1}$$

We see here how simple combinatorics provide an intrinsic and profound unification of particles and their interactions: from the two-dimensional point of view, a gauge field operator is very similar to a graviton operator, whereas the $D$-dimensional space-time properties of these operators are drastically different. An important point is that gravitons necessarily appear, which means that this kind of string theories *imply* gravity. This fact has been one of the main motivations for studying string theory.

Fermions (thought as "quarks" and "leptons") are obtained by suitably choosing the boundary conditions of the two dimensional fermions $\psi$ on $\Sigma$. These can be anti-periodic or periodic (the fields change sign or not when transported around $\Sigma$), giving rise to fermionic or bosonic fields in space-time, respectively. It turns out that whenever the theory contains space-time fermions, it must have two dimensional supersymmetry, and this is why such string theories are called *superstrings*.

It is however not so that the $D$-dimensional space-time theory must be supersymmetric, although practically all models studied so far do have supersymmetry to some degree. But this is to simplify matters and keep perturbation theory under better control, much like the study of supersymmetric Yang-Mills theory (discussed below) makes the analysis much easier as compared to non-supersymmetric gauge theories. Thus, string theory does not intrinsically predict space-time supersymmetry (in a similar spirit in which Yang-Mills theory does not predict it), though, quite misleadingly, it is often claimed so.[3]

That very simple, even non-interacting two dimensional field theories generate highly non-trivial effective dynamics in $D$-dimensional space-time, can be exhibited by looking to the perturbative effective action:




[3]An often-cited argument for low-energy supersymmetry is that perturbation theory of non-supersymmetric strings tends to be unstable due to vacuum tadpoles. However, even when starting with a supersymmetric theory, this problem will eventually appear, namely when supersymmetry is (e.g., at the weak scale) spontaneously broken. We therefore have, at any rate, to find mechanisms to stabilize the perturbation series, which means that vacuum tadpoles are no strong reason for beginning with a supersymmetric theory in the first place. See especially refs.[4, 5] for a vision how strings could be more clever than what ordinary quantum field theory intuition would makes us believe, and avoid supersymmetry (similar ideas may apply to the "hierarchy problem" as well); for a different line of thoughts, see ref.[6].

Here, $g_{\mu\nu}, A_\mu, ...$ are space-time fields that provide the background in which the strings move, and $\mathcal{L}_{2d}(\psi, X, .., g_{\mu\nu}, A_\mu, ..)$ is the lagrangian containing the two-dimensional fields, as well as the background fields which are simply coupling constants from the two dimensional point of view. The 2d fields are integrated out, and one also sums over all the possible two-dimensional world-sheets $\Sigma_\gamma$ (as well as over the boundary conditions of the $\psi(z)$). This corresponds to the well-known loop expansion of particle QFT. Note, however, that there is only one "diagram" at any given order in string perturbation theory.

The topological sum is weighted by $e^{-\phi\chi(\Sigma_\gamma)}$, where $\phi$ is the dilaton field and where the Euler number $\chi(\Sigma_\gamma) \equiv 2-2\gamma$ is the analog of the loop-counting parameter. The coupling constant for the perturbation series thus is

$$g = e^{\langle\phi\rangle},$$

which must be small in order for the perturbation series to make sense. We see here that a coupling constant is given by an *a priori* undetermined vacuum expectation value of some field, and this reflects a general principle of string theory.

In addition, the topological sum is augmented by integrals over the inequivalent shapes that the Riemann surfaces can have, and this corresponds to the momentum integrations in ordinary QFT. There may be divergences arising from degenerate shapes, but these divergences can always be interpreted as IR divergences in the space-time sense. In particular, the well-known logarithmic running of gauge couplings in four dimensional string theories arises from such IR divergences.

The absence of genuine UV divergences was another early motivation of string theory, and especially makes consistent graviton scattering possible: remember that ordinary gravity is not renormalizable and one cannot easily make sense out of graviton loop diagrams. A very important point is the origin of this well-behavedness of the string diagrams. It rests on *discrete reparametrizations* of the string-world sheets $\Sigma_\gamma$ ("modular invariance"), which have no analog in particle theory. The string "Feynman rules" are very different, and cannot even be approximated by particle QFT. String theory is therefore *more* than simply combining infinitely many particle fields, and it is this what makes a crucial difference. The whole construction is very tightly constrained: modular invariance determines the whole massive spectrum, and taking any single state away from the infinite tower of states would immediately ruin the consistency of the theory. In this sense, string theory makes infinitely many predictions.

So far, we described the ingredients that go into computing the perturbative effective action in $D$-dimensional space-time. Now we focus on the outcome. As indicated in (2), the effective action typically contains (besides matter fields) general relativity and non-abelian gauge theory, plus stringy corrections thereof. These corrections are very strongly suppressed by inverse powers of the Planck mass, $m_{\text{Planck}} \sim 10^{19}$GeV, which is the characteristic scale in the theory.

That the lagrangian of general relativity, with all its complexity, just pops out from the air (arising from eg., a non-interacting two dimensional field theory), may sound like a miracle. Of course this does not happen by accident, but is bound to come out. The important point is here that there is a special property that the relevant two dimensional theories must have for consistency, and this is *conformal invariance*. This is a quite powerful symmetry principle, which guarantees, via Ward identities, general coordinate and gauge invariance in space-time – however, only so if and only if formally $D = 10$.

As we will see momentarily, this does not imply that superstrings must live exclusively in ten dimensions, but it means that superstrings are most simply and naturally formulated in ten dimensions.

### 2.2 10d Superstrings and their compactifications

Two-dimensional field theories can have two logically independent, namely holomorphic and anti-holomorphic pieces. For obtaining $D = 10$ string theories, each of these pieces can either be of type superstring "$S$" or of type bosonic string "$B$" (with extra $E_8 \times E_8$ or $SO(32)$ gauge symmetry). By combining these building blocks in various ways,[4] plus including an additional "open" string variant, one obtains the following exhaustive list of supersymmetric strings in $D = 10$:

| Composition | Name | Gauge Group | Supersymmetry |
|---|---|---|---|
| $S \otimes \bar{S}^\dagger$ | Type IIA | $U(1)$ | non-chiral $N = 2$ |
| $S \otimes \bar{S}$ | Type IIB | - | chiral $N = 2$ |
| $S \otimes \bar{B}$ | Heterotic | $E_8 \times E_8$ | chiral $N = 1$ |
| $S \otimes \bar{B}'$ | Heterotic' | $SO(32)$ | chiral $N = 1$ |
| $(S \otimes \bar{S})/Z_2$ | Type I (open) | $SO(32)$ | chiral $N = 1$ |

Since these theories are defined in terms of 2d world-sheet degrees of freedom, which is intrinsically a perturbative concept in terms of 10d space-time physics, all we can really say is that there are five theories in ten dimensions that are different in *perturbation theory.* Their perturbative spectra are indeed completely different, their number of supersymmetries varies, and also the gauge symmetries are mostly quite different.

Now, we would be more than glad if strings would remain in lower dimensions as simple as they are in $D = 10$. However, especially string theories in $D = 4$ turn out to be much more complicated. Specifically, the simplest way to get down to four dimensions is to assume/postulate that the space-time manifold is not simply $\mathbb{R}^{10}$, but $\mathbb{R}^4 \times X_6$, where $X_6$ is some compact six-dimensional manifold. If it is small enough, then the theory looks at low energies, ie., at distances much larger than the size of $X_6$, effectively four-dimensional. This is like looking at a garden hose from a distance, where it looks one-dimensional, while upon closer inspection it turns out to be a three dimensional object.

The important good feature is that such kind of "compactified" theories are at low energies exactly of the type as the standard model of particle physics (or some supersymmetric variant thereof). That is, generically such theories will involve non-abelian gauge fields, chiral fermions and Higgs fields besides gravitation, and all of this coupled together in a (presumably) consistent and truly unified manner ! That theories that are able to do this have been found at all, is certainly reason for excitement.

But apart from this rather generic statement, more detailed predictions, like the precise gauge groups or the low-mass matter spectrum, cannot be made at present – this is the dark side of the story. The reason is that the compactification mechanism makes the theories *enormously* more complicated and rich than they originally were in $D = 10$. This is intrinsically tied to properties of the possible compactification manifolds $X_6$:

**i)** There is a large number of choices for the compactification manifold. If we want to have $N = 1$ supersymmetry in $D = 4$ from e.g. a heterotic string, then $X_6$ must be a "Calabi-Yau" space [7], and the number of such spaces is perhaps $10^4$. On top of that, one has to specify certain instanton configurations, which multiplies this number by a very large factor. *A priori*, each of these spaces (together with a choice of instanton configuration) leads to a different perturbative matter and gauge spectrum in four dimensions, and thus gives rise to a different four dimensional string theory.

[4] There are further, but non-supersymmetric theories in $D = 10$, which however do not seem to have stable ground states.

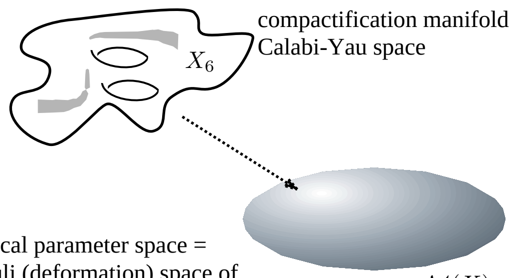



Figure 7: The parameter space of the low energy effective field theory of a string compactification is essentially given by the moduli space (deformation space) of the compactification manifold. This leads to a geometric interpretation of almost all of the parameters and couplings. Shown is that with each point in the moduli space $\mathcal{M}_X$, one associates a compactification manifold $X$ of specific shape; the shape in turn determines particle masses, Yukawa couplings etc. The picture will be refined below.

**ii)** Each of these manifolds by itself has typically a large number of parameters; for a given Calabi-Yau space, the number of parameters can easily approach the order of several hundred.

These parameters, or "moduli" determine the shape of $X_6$, and correspond physically to vacuum expectation values of scalar ("Higgs") fields, similar to the string coupling $g$ discussed above. Changing these VEVs changes physical observables at low energies, like mass parameters, Yukawa couplings and so on. They enter in the effective lagrangian as free parameters, and are not determined by any fundamental principles, at least as far is known in perturbation theory. Their values are determined by the choice of vacuum, much like the spontaneously chosen direction of the magnetization vector in a ferromagnet (which is not determined by any fundamental law either). The hope is that after breaking supersymmetry, the continuous vacuum degeneracy would be lifted by quantum corrections (which is typical for non-supersymmetric theories), so that ultimately there would be much fewer accessible vacua.

The situation is actually worse than described so far, because we have on top of points i) and ii):

**iii)** There are five theories in $D = 10$ and not just one, and a priori each one yields a different theory in four dimensions for a given compactification manifold $X_6$. If one of them would be the fundamental theory, what is then the rôle of the others ?

**iv)** There is no known reason why a ten dimensional theory wants at all to compactify down to $D = 4$; many choices of space-time background vacua of the form $\mathbb{R}^{10-n} \times X_n$ appear to be on equal footing.

All these points together form what is called the *vacuum degeneracy* problem of string theory – indeed a very formidable problem, known since a decade or so. It arises because most of the physics that is observable at low energies seems to be governed by the vacuum (zero mode) structure and not by the microscopic theory, at least as far as we can see today.

The recent progress in non-perturbative string theory does not solve the problem of the choice of vacuum state either. The progress is rather conceptual and opens up completely new perspectives on the very nature of string theory.

# 3 Duality and non-perturbative equivalences

Duality is the main new concept that has been stimulating the recent advances in supersymmetric particle [8] and string theory. Roughly speaking, duality is a map between solitonic (non-perturbative, non-local, "magnetic") degrees of freedom, and elementary (perturbative, local, "electric") degrees of freedom. Typically, duality transformations exchange weak and strong-coupling physics and act on coupling constants like $g \to 1/g$. They are thus intrinsically of quantum nature.

Duality symmetries are most manifest in supersymmetric theories, because in such theories perturbative loop corrections tend to be suppressed, due to cancellations between bosonic and fermionic degrees of freedom. Otherwise, observable quantities get so much polluted by radiative corrections, that the more interesting non-perturbative features cannot be easily made out.

More precisely, certain quantities (eg., Yukawa couplings) in a supersymmetric low energy effective action are protected by non-renormalization theorems, and those quantities are typically *holomorphic* functions of the massless fields. As a consequence, this allows to apply methods of complex analysis, and ultimately of algebraic geometry [9], to analyze the physical theory. Such methods (where they can be applied) turn out to be much more powerful then traditional techniques of quantum field theory, and this was the technical key to the recent developments.

A typical consequence of the holomorphic structure is a continuous vacuum degeneracy, arising from flat directions in the effective potential. The non-renormalization properties then guarantee that this vacuum degeneracy is not lifted by quantum corrections, so that supersymmetric theories often have continuous quantum moduli spaces $\mathcal{M}$ of vacua. In string theory, as mentioned above, these parameter spaces can be understood geometrically as moduli spaces of compactification manifolds.

## 3.1 A gauge theory example

One of the milestones in the past few years was the solution of the (low energy limit of) $N = 2$ supersymmetric gauge theory in four dimensions [10, 11]. Important insights that go beyond conventional particle field theory have been gained by studying this model, and this is why we briefly touch it here. In fact, even though this model is an ordinary gauge theory, the techniques stem from string theory, which demonstrates that string ideas can be useful also for ordinary field theory.

Without going too much into the details, simply note that $N = 2$ supersymmetric gauge theory (here for gauge group $G = SU(2)$) has a moduli space $\mathcal{M}$ that is spanned by roughly the vacuum expectation value of a complex Higgs field $\phi$. The relevant holomorphic quantity is the effective, field dependent gauge coupling $g(\phi)$ (made complex by including the theta-angle in its definition). Each point in the moduli space corresponds to a particular choice of the vacuum state. Moving around in $\mathcal{M}$ will change many properties of the theory, like the value of the effective gauge coupling $g(\phi)$ or the mass spectrum of the physical states. An important aspect is that there are special regions in the moduli space, where the effective theory behaves specially, ie., becomes *singular*. This is depicted in Fig.8.

More precisely, there are two different types of such singular regions. Near $\langle\phi\rangle \to \infty$, the gauge theory is weakly coupled since the effective gauge coupling becomes arbitrarily small: $g(\phi) \to 0$. In this semi-classical region, non-perturbative effects are strongly suppressed and the perturbative definition of the theory is arbitrarily good. That is, the

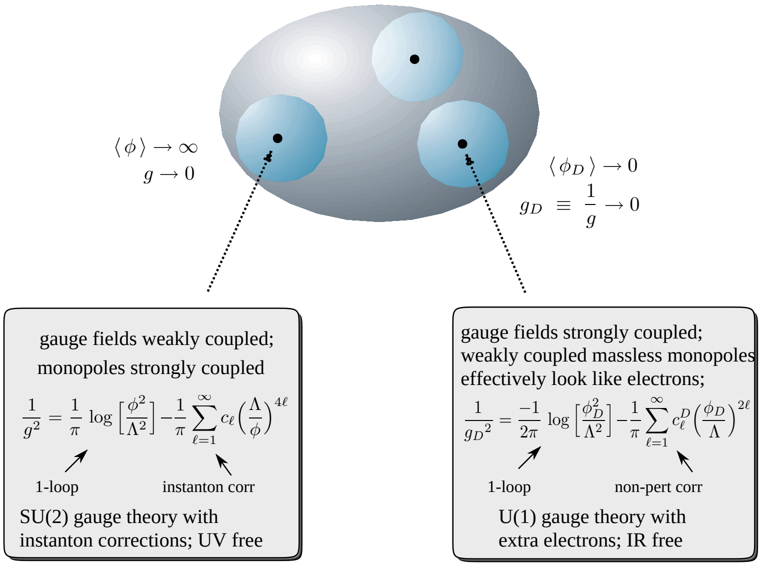



Figure 8: The exact quantum moduli space of $N = 2$ supersymmetric $SU(2)$ Yang-Mills theory has one singularity at weak coupling and two singularities in the strong coupling region. The latter are caused by magnetic monopoles becoming massless for the corresponding vacuum values of the Higgs field. One can go between the various regions by analytic continuation, ie., by resumming the non-perturbative instanton corrections in terms of suitable variables.

instanton correction sum to the left in Fig.8 gives a negligible contribution as compared to the logarithmic one-loop correction (further higher loop corrections are forbidden by the $N = 2$ supersymmetry). Furthermore, the solitonic magnetic monopoles that exist in the theory become very heavy and effectively decouple.

However, when we now start moving in the moduli space $\mathcal{M}$ away from the weak coupling region, the non-perturbative instanton sum to the left in Fig.8 will less and less well converge, and the original perturbative definition of the theory will become worse and worse. When we are finally close to one of the other two singularities in Fig.8, the original perturbative definition is blurred out and does not make sense any more. The problem is thus how to suitably *analytically continue* the complex gauge coupling outside the region of convergence of the instanton series. The way to do this is to resum the instanton series in terms of another variable, $\phi_D$, to yield another expression for the gauge coupling that is well defined near $1/g \to 0$. That is, there is a "dual" Higgs field, $\phi_D$, in terms of which the dual gauge coupling, $g_D(\phi_D)$, makes sense in the strong coupling region of the parameter space $\mathcal{M}$. Indeed, $\phi_D$ becomes small in this region, so that the infinite series for the dual coupling $g_D$ to the right in Fig.8 converges very well.

The important point to note here is that the perturbative physics in the strong coupling region is completely different as compared to the perturbative physics in the weak coupling region that we started with ! At weak coupling, we had a non-abelian $SU(2)$ gauge theory, while at strong coupling we have now an abelian $U(1)$ gauge theory plus some extra massless matter fields ("electrons"). But the latter only manifest themselves as elementary fields if we express the theory in terms of the appropriate dual variables; in the original variables, these "electrons" that become light at strong coupling, are nothing but some of the solitonic magnetic monopoles that were originally very massive in the weak coupling region.

All this said, we still do not know how to actually solve the theory and determine all the

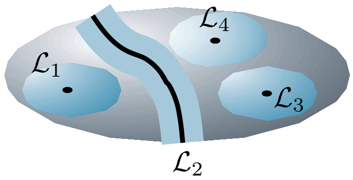



Figure 9: The moduli space of a generic supersymmetric theory is covered by coordinate patches, at the center of each of which the theory is weakly coupled when choosing suitable local variables. A local effective lagrangian exists in each patch, representing a particular perturbative approximation. None of such lagrangians is more fundamental than the other ones.

unknown non-perturbative instanton coefficients $c_\ell$, $c_\ell^D$ in Fig.8. A direct computation would be beyond what is currently possible with ordinary field theory methods. The insight of Seiberg and Witten [10] was to realize that the *patching together of the known perturbative data in a globally consistent way* is so restrictive that it fixes the theory, and ultimately gives explicit numbers for the instanton coefficients $c_\ell$ and $c_\ell^D$. This shows that sometimes much can be gained by not only looking to a theory at some given fixed background parameters, but rather by looking to a whole family of vacua, i.e., to global properties of the moduli space.

### 3.2 The message we can abstract from the field theory example

The lesson is that one and the same physical theory can have many perturbative descriptions. These may look completely different, and can involve different gauge groups and matter fields. There is in general no absolute notion of what would be weak or strong coupling; rather what we call weak coupling or strong coupling, or an elementary or a solitonic field, depends to some extent on the specific description that we use. Which description is most suitable, and which physical degrees of freedom will be light or weakly coupled (if any at all), depends on the region of the parameter space we are looking at.

More mathematically speaking, an effective lagrangian description makes sense only in *local coordinate patches* covering the parameter space $\mathcal{M}$ – see Fig.9. These describe different perturbative approximations of the same theory in terms of different weakly coupled physical local degrees of freedom (eg, electrons or monopoles). No particular effective lagrangian is more fundamental than any other one. In the same way that a topologically non-trivial manifold cannot be covered by just one set of coordinates, there is in general no globally valid description of a family of physical theories in terms of a single lagrangian.

It is these ideas that carry over, in a refined manner, to string theory and thus to grand unification; in particular, they have us rethink concepts like "distinguished fundamental degrees of freedom". String moduli spaces will however be much more complex than those of field theories, due to the larger variety of possible non-perturbative states.

### 3.3 $P$-branes and non-perturbative states in string theory

String compactifications on manifolds $X$ are not only complex because of the large moduli spaces they generically have, but also because the spectrum of physical states becomes vastly more complicated. In fact, when going down in the dimension by compactification, there is a dramatic *proliferation of non-perturbative states*.

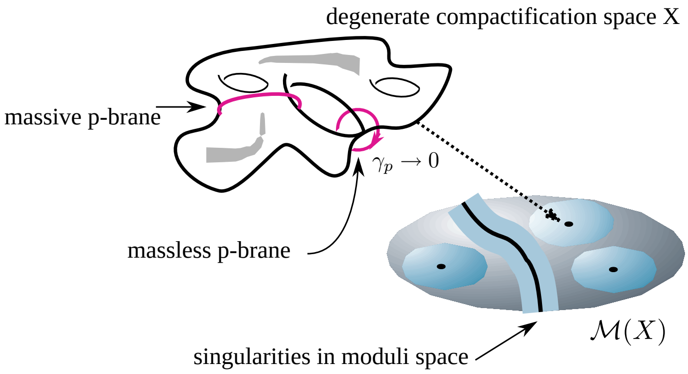



Figure 10: Non-perturbative particle-like states arise from wrapping of $p$-branes around $p$-dimensional cycles $\gamma_p$ of the compactification manifold $X$. These are generically very massive, but can become massless in regions of the moduli space where the $p$-cycles shrink to zero size. The singularities in the moduli space are the analogs of the monopole singularities in Fig.8.

The reason is that string theory is not simply a theory of strings: there exist also higher dimensional extended objects, so called "$p$-branes", which have $p$ space and one time dimensions (in this language, strings are 1-branes, membranes 2-branes etc; generically, $p = 0, 1, ..., 9$). Besides historical reasons, string theory is only called so because strings are typically the lightest of such extended objects. In the light of duality, as discussed in the previous sections, we know that there is no absolute distinction between elementary or solitonic objects. We thus expect $p$-branes to play a more important rôle than originally thought, and not necessarily to just represent very heavy objects that decouple at low energies.

More specifically, such $(p + 1)$ dimensional objects can wrap around $p$-dimensional cycles $\gamma_p$ of a compactification manifold, to effectively become particle-like excitations in the lower dimensional (say, four dimensional) theory. These extra solitonic states are analogs of the magnetic monopoles that had played an important rôle in the $N = 2$ supersymmetric Yang-Mills theory. Since such monopoles can be light and even be the dominant degrees of freedom for certain parameter values, we expect something similar for the wrapped $p$-branes in string compactifications. In fact, the volumina of $p$-dimensional cycles $\gamma_p$ of $X_n$ depend on the deformation parameters, and there are singular regions in the moduli space where such cycles shrink to zero size ("vanishing cycles") – see Fig.10. Concretely, if a $p$-dimensional cycle $\gamma_p$ collapses, then a $p$-brane wrapped around $\gamma_p$ will give a massless state in $D = 10 - n$ dimensional space-time [12]. This is because the mass formula for the wrapped brane involves an integration over $\gamma_p$:

$$m_{p-\mathrm{brane}}{}^2 \;=\; |\int_{\gamma_p} \Omega\,|^2 \;\longrightarrow\; 0 \qquad \text{if } \gamma_p \to 0\ .$$

The larger the dimension of the compactification manifold $X_n$ is (and the lower the space-time dimension $D = 10 - n$), the more complicated the topology of it will be, ie., the larger the number of "holes" around which the branes can wrap. For a six dimensional Calabi-Yau space $X_6$, there will be generically an abundance of extra non-perturbative states that are not seen in traditional string perturbation theory. These can show up in $D = 4$ as ordinary gauge or matter fields. It is the presence of such non-perturbative, potentially massless states what is the basis for many non-trivial dual equivalences of string theories.

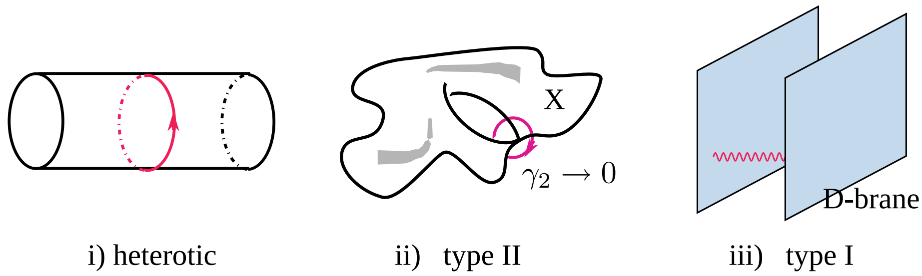



Figure 11: Different geometries can parametrize one and the same physical situation. Shown here are three dual ways to represent an $SU(2)$ gauge field and the associated Higgs mechanism.

When going back to ten dimensions by making the compactification manifold very large ("decompactification"), these states become arbitrarily heavy and eventually decouple. In this sense, a string model has after compactification many more states that were not present in ten dimensions before. In the compactified model, the non-perturbative spectrum is very finely tuned: in an analogous way that removing a perturbative string state from the spectrum would destroy modular invariance (which is a global property of the 2d world-sheet) and thus would ruin perturbative consistency, taking out a non-perturbative state would destroy duality symmetries (which are a global property of the compactification manifolds): it would make the global behavior of the theory over the moduli space inconsistent.

### 3.4 Stringy geometry

We have seen in section 2.1 that $N = 2$ supersymmetric Yang-Mills theory is sometimes better described in terms of dual "magnetic" variables, namely when we are in a region of the moduli space where certain magnetic monopoles are light. In this dual formulation, the originally solitonic monopoles look like ordinary elementary, perturbative degrees of freedom ("electrons"). Analogously, dual formulations exist also for string theories, in which non-perturbative solitons are described in terms of weakly coupled "elementary" degrees of freedom. It was one of the breakthroughs when it was realized how this exactly works: the relevant objects dual to certain solitonic states are special kinds of ($p$-)branes, so-called "$D(p$-)branes" [13]. Due to the many types of $p$-branes on the one hand, and the large variety of possible singular geometries of the manifolds $X$ on the other, the general situation is however very complicated. It is easiest to describe it in terms of typical examples.

For instance, massless or almost massless non-abelian gauge bosons $W^{\pm}$ belonging to $SU(2)$ can be equivalently described in a number of dual ways (see Fig.11):
**i)** In the heterotic string compactified on some higher dimensional torus, a massless gauge boson is represented by a fundamental heterotic string wrapped around part of the torus, with a certain radius (say $R = 1$ in some units; changing the orientation of the string maps $W^+ \leftrightarrow W^-$).This is a perturbative description, since it involves an elementary string. If the radius deviates from $R = 1$, the gauge field gets a mass, providing a geometrical realization of the Higgs mechanism.
**ii)** In the compactified type IIA string, the gauge boson arises from wrapping a 2-brane around a collapsed 2-cycle $\gamma_2$ of $X$. This is a non-perturbative formulation in terms of string theory. If $\gamma_2$ does not quite vanish, the gauge field retains some non-zero mass, thereby realizing the Higgs mechanism in a different manner.

**iii)** In the type I string model, an $SU(2)$ gauge boson is realized by an open string stretched between two flat $D$-branes. This is another perturbative formulation of the Higgs mechanism. The mass of the gauge boson is proportional to the length of the open string, and thus vanishes if the two $D$-branes move on top of each other.

We thus see that very different mathematical geometries can represent the identical physical theory, here the $SU(2)$ Higgs model – these geometries really should be *identified* in string theory, since there is no possible measurement one could possibly make to distinguish them ! This provides a special example of a more general concept, which is about getting better and better understood: "stringy geometry". In stringy geometry, certain mathematically different geometries are regarded as equivalent, and just seen as different choices of coordinates, or parametrizations of one and the same abstract space of string theories. The underlying physical idea is that while in ordinary geometry point particles are used to measure properties of space-time, in stringy geometry one augments this by string and other $p$-brane probes. It is the wrappings and stretchings that these extra objects are able to do that can wash out the difference between topologically and geometrically distinct manifolds.

In effect, a string theory of some type "$A$" when compactified on some manifold $X_A$, can be dual, ie., quantum equivalent, to another string theory "$B$" on some manifold $X_B$ – and this even if $A$, $B$ and/or $X_A$ and $X_B$ are profoundly different (a prime example is given by the type IIA string compactified on a "$K3$ manifold"[5], which is indistinguishable from the heterotic string compactified on the four-dimensional torus, $T_4$ [14]). Again, all what matters is that the full non-perturbative theories coincide, while there is no need for the perturbative approximations to be even remotely similar.

# 4 The Grand Picture

## 4.1 Dualities of higher dimensional string theories

We are now prepared to go back to ten dimensions and revisit the five perturbatively defined string models of section 1.2. In view of the remarks of the preceding sections concerning the irrelevance of perturbative concepts, we will now find it perhaps not too surprising to note that these five theories are really nothing but different approximations of just one theory. In complete analogy to what we said about $N = 2$ supersymmetric gauge theory in section 2.1, they simply correspond to certain choices of preferred "coordinates" that are adapted to specific parameter regions.

Although these facts can be stated in such simple terms, they are so non-trivial that it took more than a decade to discover them. It can now be explicitly shown that by compactifying any one of the five theories on a suitable manifold, and then un-compactifying it in another manner, one can reach any other of the five theories in a continuous way.

It was a great surprise when the strong-coupling limit of the type IIA string was investigated [15]: it turned out that in this limit, certain non-perturbative "$D0$-branes" form a continuous spectrum and effectively generate an extra 11th dimension. That is, at ultra strong coupling the ten-dimensional type IIA string theory miraculously gains 11-dimensional Lorentz invariance, and the low-energy theory turns into $D = 11$ supergravity. This was especially stunning because eleven dimensional supergravity is not a low energy limit of a string theory, but rather seems to be related to supermembranes. In other words, non-perturbative dualities take us beyond string theory !

[5] $K3$ denotes the (unique) four-dimensional analog of a Calabi-Yau space.

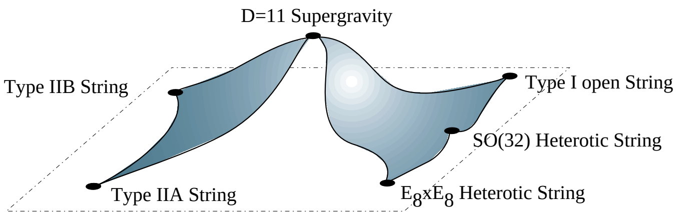



Figure 12: Moduli space in ten and eleven dimensions. Its singular asymptotic regions correspond to the five well-known supersymmetric string theories in $D = 10$, plus an eleven dimensional $M$-theory. The vertical direction roughly reflects the space-time dimension.

So what we have are not five but six 10- or 11-dimensional approximations, or local coordinate patches on some moduli space – see Fig.12. But to what entity are these theories approximations ? Do we have here the moduli space of some underlying microscopic theory ? Indeed there is a candidate for such a theory, dubbed "$M$-theory" [16, 17]. Its low energy limit does give $D = 11$ supergravity, and simple compactifications of it give all of the five string theories in $D = 10$. It may well be that $M$-theory, currently under intense investigation, holds the key for a detailed understanding of non-perturbative string theory. However, we will not discuss any further details here, we will rather return to the lower dimensional theories.

### 4.2 Quantum moduli space of four-dimensional strings

Fig.12 shows only a small piece of a much more extended moduli space, namely only the piece that describes higher dimensional theories. These are relatively simple and there is only a small number of them. As discussed above, the lower dimensional theories obtained by compactification are much more intricate.

Among the best-investigated string theories in four dimensions are the ones with $N = 2$ supersymmetry – these are the closest analogs of the $N = 2$ gauge theory that is so well understood. They can be obtained by compactifications of type IIA/B strings on Calabi-Yau manifolds $X_6$, or dually, by compactifications of heterotic strings on $K3 \times T_2$. Since there are roughly $10^4$ of such Calabi-Yau manifolds $X_6$ known, the complete $D = 4$ string moduli space will have roughly $10^4$ components, each with typical dimension 100 (keeping in mind that there can be non-trivial identifications between parts of this moduli space)– see Fig.13. We see that the moduli space is drastically more complicated as it is either for the high-dimensional theories, or for the $N = 2$ gauge theory. Each of the $10^4$ components typically has several ten or eleven dimensional decompactification limits, so that one should imagine very many connections between the upper and lower parts of Fig.13 (indicated by dashed lines).

An interesting fact is that (practically[6]) all of the known $10^4$ families of perturbative $D = 4$ string vacua are connected by non-perturbative extremal transitions. To understand what we mean by that, we simply follow a path in the moduli space as indicated in Fig.13, starting somewhere in the interior of a blob. The massive spectrum will continuously change, and when we hit singularities, extra massless states appear and perturbation theory breaks down.

It can in particular happen that a non-perturbative massless Higgs field appears, to which we can subsequently give a vacuum expectation value to deform the theory in a

[6]Strictly speaking, there seem some further, disconnected components to exist.

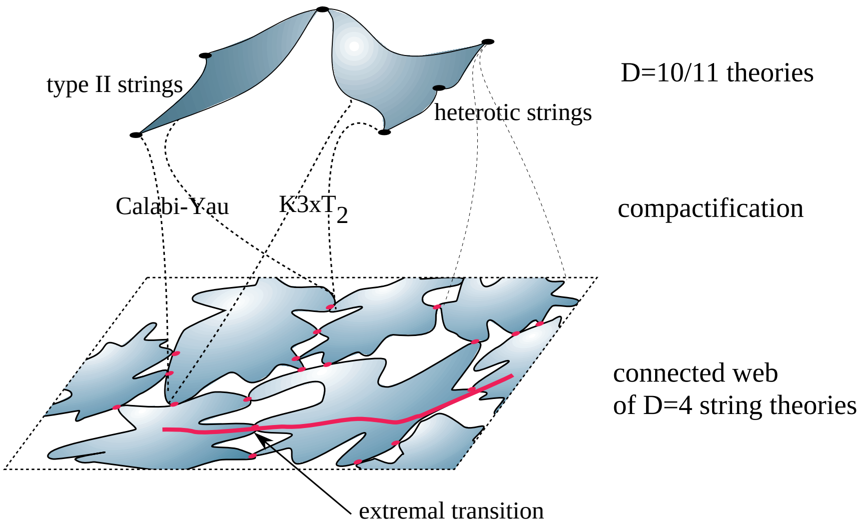



Figure 13: Rough sketch of the space of $N = 2$ supersymmetric string theories in four dimensions. In general, a given region in the 4d moduli space can be reached in several dual ways via compactification of the higher dimensional theories. Each of the perhaps $10^4$ components corresponds typically to a 100 dimensional family of perturbative string vacua, and represents the moduli space of a single Calabi-Yau manifold (like the one shown in Fig.7). Non-perturbatively, these vacua turn out to be connected by extremal transitions and thus form a continuous web.

direction that was not visible before. In this way, we can leave the moduli space of a single perturbative string compactification, and enter the moduli space of another one. Thus, non-perturbative extra states can glue together different perturbative families of vacua in a physically smooth way [12]. It can be proven that one can connect in this manner all of the roughly $10^4$ known components that were previously attributed to different four dimensional string theories. In other words, the full non-perturbative quantum moduli space of $N = 2$ supersymmetric strings seems to form *one* single entity.

This is not much of a practical help for solving the vacuum degeneracy problem, but it is conceptually satisfying: instead having to choose between many four dimensional string theories, each one equipped with its own moduli space, we really have just one theory with however very many facets.

This as far as $N = 2$ supersymmetric strings in four dimensions are concerned – the situation is still more complicated for the phenomenologically important $N = 1$ supersymmetric string theories, about which much less is known. The main novel features that can be addressed in these theories are fermion chirality, and supersymmetry breaking. It seems that certain aspects of these theories are best described by choosing still another dual formulation, called "$F$-theory" [18]. This is a construction formally living in twelve dimensions, and whether it is simply a trick to describe certain features in an elegant fashion, or whether it is a honest novel theory in its own, remains to be seen.

## 5 "Theoretical experiments"

How can we convince ourselves that the considerations of the previous sections really make sense ? Clearly, all what we can do for the foreseeable future to test these ideas are consistency checks. Such checks can be highly non-trivial, from a formal as well as from

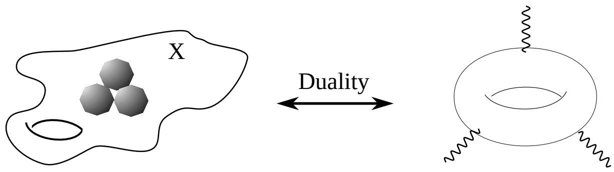



Figure 14: Counting spheres in a Calabi-Yau space $X$ in type IIA string theory, leads ultimately to the same expressions for certain low energy couplings as a standard one-loop computation in the heterotic string.

a physical point of view. So far, numerous qualitative and quantitative tests have been performed, and not a single test on the dualities has ever failed !
To give some flavor, let us recapitulate a few characteristic checks:

**i)** *Complicated perturbative corrections* can be computed and compared for string models that are dual to each other. In all cases, the results perfectly agree. Consider for example certain 3-point couplings $\kappa$ in $N = 2$ supersymmetric string compactifications. As mentioned before, such theories can be obtained either from type II strings on Calabi-Yau manifolds $X_6$, or dually from heterotic string compactifications on $K3 \times T_2$. In the type II formulation, these couplings can be computed (via “mirror symmetry” [19]) by counting world-sheet instantons (embedded spheres) inside the Calabi-Yau space. Concretely, in a specific model the result takes the following explicit form:

$$\kappa \;=\; \frac{i}{2\pi}\frac{E_4(T)E_4(U)E_6(U)(E_4(T)^3 - E_6(T)^2)}{E_4(U)^3 E_6(T)^2 - E_4(T)^3 E_6(U)^2}\ ,$$

in terms of certain modular functions $E_4$, $E_6$ depending on moduli fields $T, U$. The very same expression can be obtained also in a completely different manner, namely by performing a standard one-loop computation in the dual heterotic string model – in precise agreement with the postulated string duality; see Fig.14. Many similar tests, also involving higher loops and gravitational couplings, have been shown to work out as well.

**ii)** *State count in black holes.* This is a highly non-trivial physics test. The issue is to compute the Bekenstein-Hawking entropy $S_{BH}$ (=area of horizon) of an extremal (or near-extremal) black hole. Strominger and Vafa [20] considered the particular case of an extremal $N = 4$ supersymmetric black hole in $D = 5$, where one knows that

$$S_{BH} \;=\; 2\pi\sqrt{\frac{q_f q_h}{2}}\ ,$$

where $q_f$, $q_h$ are the electric and axionic charges of the black hole. The idea is to use string duality to represent a large semi-classical black hole in terms of a type IIB string compactification on $K3 \times S^1$. This eventually boils down to a 2d sigma model on the moduli space of a gas of $D0$-branes on $K3$. Counting states in this model indeed exactly reproduces the above entropy formula for large charges.

This test (and many refinements of it) does not only add credibility to the string duality claims from a new perspective, but also tells that string theory seems to be *complete*, in that there are no missing degrees of freedom that we might have been overlooking – any other conceivable theory of quantum gravity better comes up with the same count of relevant microscopic states (and so is likely to be equivalent to the string theory anyway).

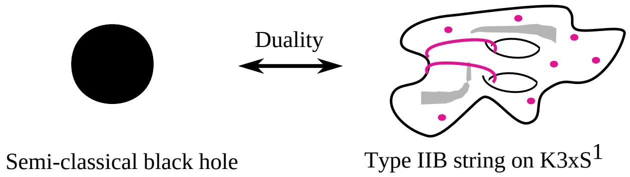



Figure 15: Counting black hole microstates, by mapping the problem to a gas of $D$-branes on a $K3$ manifold.

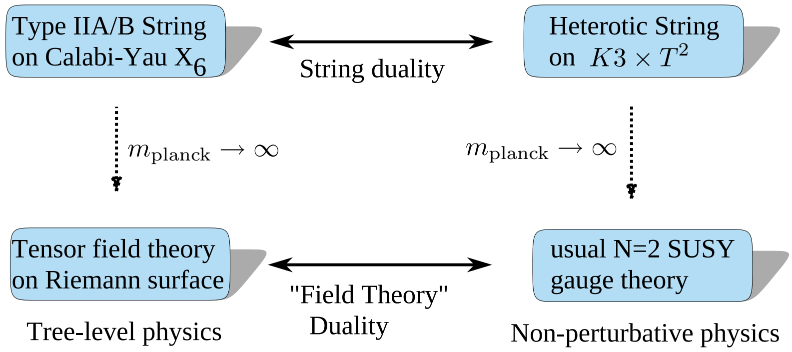



Figure 16: Recovering Seiberg-Witten theory from string duality. In the field theory limit, one sends the Planck mass to infinity in order to decouple gravity and other effects that are not important. A remnant of the string duality survives, which is a duality between the standard formulation of the gauge theory, and a novel one that was not known before. It gives a physical interpretation of the geometry underlying the non-perturbative solution of the $N = 2$ gauge theory.

**ii)** *Recovering exact non-perturbative field theory results.* One can derive the exact solution of the $N = 2$ supersymmetric gauge theory described in section 2.1 as a consequence of string duality. The point is that duality often maps classical into quantum effects and vice versa. This makes it for example possible [21] to obtain with a *classical* computation in the compactified type II string, certain non-perturbative quantum results for the compactified heterotic string. In particular, counting world-sheet instantons similarly as in point i) above, and suitably decoupling gravity (see Fig.16), allows to exactly reproduce [22] the non-perturbative effective gauge coupling function $g(\phi)$ of Fig.8.

This is very satisfying, as it gives support to both the underlying string duality and the solution of the $N = 2$ gauge theory.[7] Indeed, while the basic heterotic-type II string duality [14] and the Seiberg-Witten theory [10] have been found independently from each other around the same time, their mutual compatibility was shown only later.

## 6 Outlook

In principle, it might have been that, for example, the string-derived non-perturbative results on the gauge theory were different as compared to the results obtained by Seiberg and Witten, and this would have obviously been quite a disaster. But no, it did come out right, as it did in all the many other instances were dualities were checked. It is this clearly visible, *convergent evolution* of a priori separate physical concepts, besides overwhelming “experimental” evidence, what gives the string theorists confidence in the

[7] The techniques for obtaining exact non-perturbative results for ordinary field theories from string duality are not limited to only reproducing results that one already knows – in fact, they have opened the door for deriving new results for a whole variety of novel quantum theories in various dimensions.

validity of their ideas.

Indeed every time a new mosaic stone is found, it happens to fit perfectly to the other mosaic stones ! That this process seems to continue all the time and a clearer and clearer picture emerges, is so non-trivial that there cannot be any doubt that the whole building of physical ideas and concepts is completely self-consistent and makes sense as a physical model. In view of the many non-trivial consistency constraints that are fulfilled, it is most likely that there is simply no room for a "different" consistent theory; in other words, it is likely that what we have found is the complete space of all possible consistent quantum theories that include gravity, and string theory may perhaps be viewed as one way of efficiently parametrizing it (in certain regions of its parameter space). Whether this self-consistent model, or rather one particular point in its huge parameter space, does actually describe our world, is another question, a question that we may not be able to decide for a long time. At any rate, for the moment there seems to be no other option than trying to improve our understanding of the structure that we see emerging.

## References[8]


1 B. Greene, *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory,* Norton & Company 1999, ISBN 0393046885.

2 For introductory textbooks, see e.g.:
M. Green, J. Schwarz and E. Witten, *Superstring Theory*, Vol. 1-2, Cambridge University Press 1987;
D. Lüst and S. Theisen, *Lectures on String Theory*, Lecture Notes in Physics 346, Springer 1989.
J. Polchinski, *String Theory: An Introduction to the Bosonic String (Cambridge Monographs on Mathematical Physics) Vol. 1,* and *String Theory: Superstring Theory and Beyond (Cambridge Monographs on Mathematical Physics , Vol 2), Cambridge University Press 1998, ISBN 0521633036 and 0521633044.*
For more recent review papers, see e.g.:
J. Polchinski, *What is string theory?,* lectures presented at the 1994 Les Houches Summer School, `hep-th/9411028`; *String duality: A Colloquium,* Rev. Mod. Phys. **68** (1996) 1245-1258, `hep-th/9607050`;
K. Dienes, *String theory and the path to unification: A Review of recent developments,* Phys. Rept. **287** (1997) 447-525, `hep-th/9602045`;
C. Vafa, *Lectures on strings and dualities,* lectures at ICTP Summer School 1996, `hep-th/9702201`;
E. Kiritsis, *Introduction to superstring theory,* `hep-th/9709062`;
S. Mukhi, *Recent developments in string theory: A Brief review for particle physicists,* `hep-ph/9710470`.
See also superstring web sites, eg., http://www.physics.ucsb.edu/~jpierre/strings.

3 See eg., N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The Hierarchy problem and new dimensions at a millimeter,* Phys. Lett. **B429** 263 (1998), `hep-ph/9803315`.

4 G. Moore, *Atkin-Lehner symmetry,* Nucl. Phys. **B293** (1987) 139.

5 K. Dienes, as cited in ref. [2].


[8] hep-th/yymmnnn refers to the web address: http://xxx.lanl.gov/abs/hep-th/yymmnnn.

6 E. Witten, *Strong coupling and the cosmological constant,* Mod. Phys. Lett. **A10** (1995) 2153-2156, `hep-th/9506101`.

7 For a review, see: B. Greene, *String theory on Calabi-Yau manifolds,* lectures at TASI-96 Summer School, `hep-th/9702155`.

8 For a review on supersymmetric gauge theories, see:
K. Intriligator and N. Seiberg, *Lectures on supersymmetric gauge theories and electric - magnetic duality,* Nucl. Phys. Proc. Suppl. **45BC** (1996) 1-28, `hep-th/9509066`.

9 For a review emphasizing mathematical aspects of Duality, see: C. Vafa, *Geometric physics,* `hep-th/9810149`.

10 N. Seiberg and E. Witten, *Electric - magnetic duality, monopole condensation, and confinement in N=2 supersymmetric Yang-Mills theory, Nucl. Phys.* B426 (1994) 19, `hep-th/9407087`; *Monopoles, duality and chiral symmetry breaking in N=2 supersymmetric QCD, Nucl. Phys.* B431 (1994) 484, `hep-th/9408099`.

11 For reviews see, for example:
L. Alvarez-Gaumé and S. Hassan, *Introduction to S-Duality in N=2 supersymmetric Gauge Theories: A pedagogical review of the Work of Seiberg and Witten,* Fortsch. Phys. **45** (1997) 159-236, `hep-th/9701069`;
W. Lerche, *Introduction to Seiberg-Witten Theory and its Stringy Origin*, Nucl. Phys. Proc. Suppl. *55B* (1996) 83, `hep-th/9611190`;
M. Peskin, *Duality in Supersymmetric Yang-Mills Theory,* lectures at TASI-96 Summer School, `hep-th/9702094`.

12 A. Strominger, *Massless black holes and conifolds in string theory, Nucl. Phys.* B451 (1995) 96, `hep-th/9504090`;
for a review, see: A. Strominger, *Black Hole Condensation and Duality in String Theory,* `hep-th/9510207`.

13 J. Polchinski, *Dirichlet-Branes and Ramond-Ramond Charges,* Phys. Rev. Lett. **75** (1995) 4724-4727, `hep-th/9510017`;
for a review, see: J. Polchinski, *Tasi Lectures on D-Branes,* TASI-96 Summer School, `hep-th/9611050`.

14 C. Hull and P. Townsend, *Unity of superstring dualities, Nucl. Phys.* B438 (1995) 109, `hep-th/9410167`.

15 E. Witten, *Some Comments on String Dynamics,* `hep-th/9507121`.

16 T. Banks, W. Fischler, S. Shenker and L. Susskind, *M-Theory as a Matrix Model: A Conjecture,* Phys. Rev. **D55** (1997) 5112-5128, `hep-th/9610043`.

17 For reviews, see e.g.:
A. Bilal, *M(atrix) Theory: a Pedagogical Introduction,* `hep-th/9710136.`;
T. Banks, *Matrix Theory,* Trieste Spring School 1997, `hep-th/9710231`.

18 C. Vafa, *Evidence for F-theory,* Nucl. Phys. **B469** (1996) 403-418, `hep-th/9602022`.

19 S. Yau (ed.), *Essays on Mirror Manifolds*, International Press 1992; B. Greene (ed.), *Mirror symmetry II*, AMS/IP Studies in advanced mathematics vol 1, 1997.

20 A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy,* Phys. Lett. **B379** (1996) 99-104, `hep-th/9601029`.

21 S. Kachru and C. Vafa, *Exact Results for N=2 Compactifications of Heterotic Strings,* Nucl. Phys. **B450** (1995) 69-89, `hep-th/9505105`;
S. Ferrara, J. Harvey, A. Strominger and C. Vafa, *Second quantized Mirror Symmetry,* Phys. Lett. **B361** (1995) 59-65, `hep-th/9505162`.

22 S. Kachru, A. Klemm, W. Lerche, P. Mayr and C. Vafa, *Nonperturbative Results on the Point Particle Limit of N=2 Heterotic String Compactifications,* Nucl. Phys. **B459** (1996) 537-558, `hep-th/9508155`.